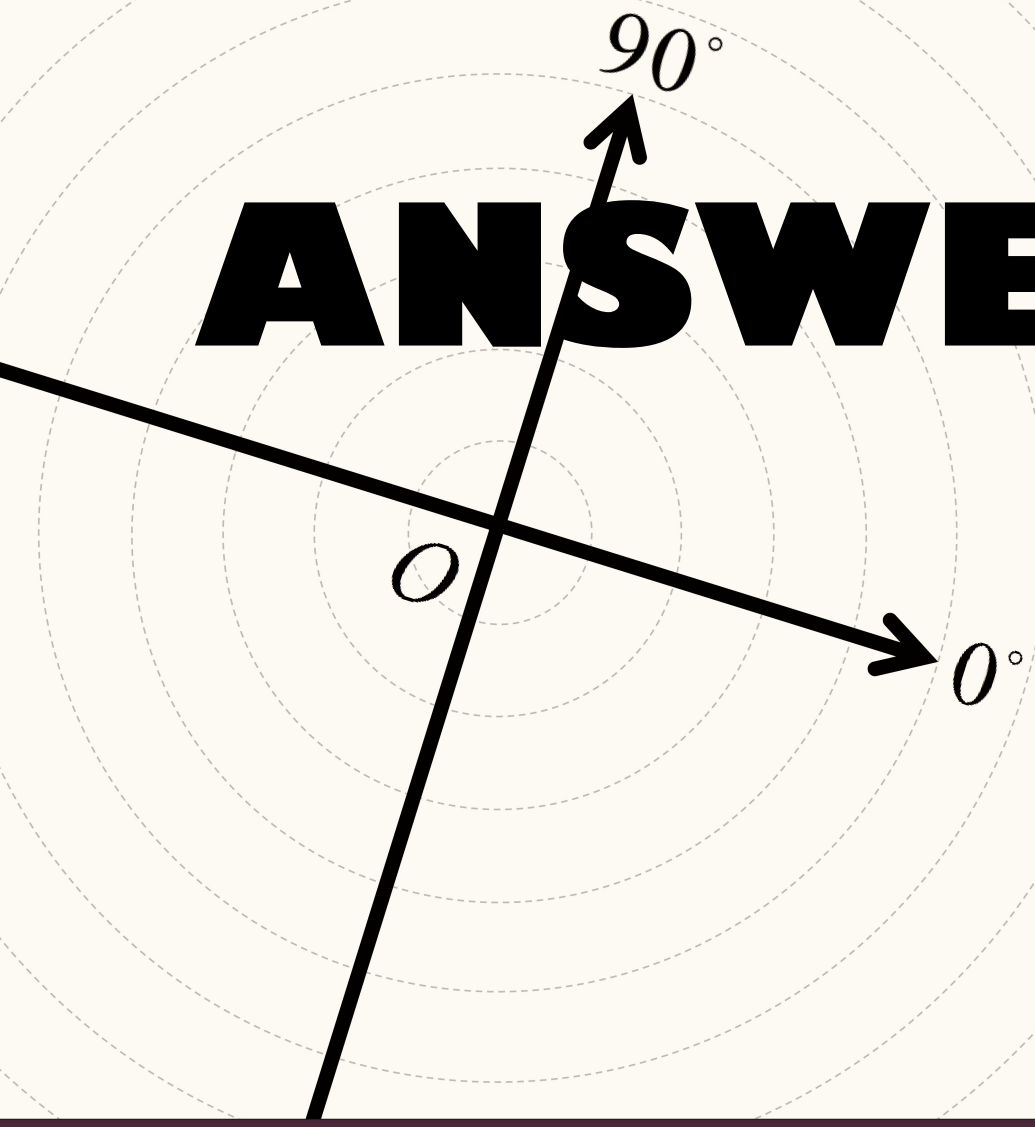
YOUR PRACTICE SET

APPLICATIONS AND INTERPRETATION FOR IBDP MATHEMATICS

Book 1



ANSWERS

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- Common Topics for both SL and HL students
- 100 Examples + 400 Intensive Exercises
- 400 Short Questions + 100 Structured Questions
- Skills on GDC

SE PRODUCTION LIMITED

Chapter 1 Solution

Exercise 1

1. (a) The required circumference

$= 1730 \times \pi$ (M1) for correct formula

$= 5434.955291$

$= 5.43 \times 10^3$ cm A1 N2

[2]

(b) The required area

$= \left(\frac{1730}{2}\right)^2 \times \pi$ (M1) for correct formula

$= 2350618.163$

$= 2.35 \times 10^6$ cm^2 A1 N2

[2]

2. (a) The required length of hypotenuse

$= \sqrt{3348^2 + 14880^2}$ (M1) for correct formula

$= 15252$

$= 1.53 \times 10^4$ cm A1 N2

[2]

(b) The required area

$= \frac{1}{2} \times 3348 \times 14880$ (M1) for correct formula

$= 24909120$

$= 2.49 \times 10^7$ cm^2 A1 N2

[2]

3. (a) The required height

$= \dfrac{22489932}{5476}$ (M1) for correct formula

$= 4107$

$= 4.11 \times 10^3$ cm A1 N2

[2]

(b) The required length of diagonal

$= \sqrt{4107^2 + 5476^2}$ (M1) for correct formula

$= 6845$

$= 6.85 \times 10^3$ cm A1 N2

[2]

4. (a) The required base length

$= \dfrac{331320000}{8283} \times 2$ (M1) for correct formula

$= 80000$

$= 8 \times 10^4$ cm A1 N2

[2]

(b) The required length of hypotenuse

$= \sqrt{80000^2 + 8283^2}$ (M1) for correct formula

$= 80427.65749$

$= 8.04 \times 10^4$ cm A1 N2

[2]

Chapter 2 Solution

Exercise 2

1. (a) 2.8125 — A1 N1 [1]

(b) 2.81 — A1 N1 [1]

(c) $2.805 \le B < 2.815$ — A2 N2 [2]

(d) The percentage error

$= \left| \frac{2.84 - 2.8125}{2.8125} \right| \times 100\%$ — (A1) for correct substitution

$= 0.977777778\%$

$= 0.978\%$ — A1 N2 [2]

2. (a) The perimeter

$= 5.278 + 4.812 + 4.118 + 3.756$ — (M1) for valid approach

$= 17.964 \text{ cm}$ — A1 N2 [2]

(b) The upper bound is 5.35 cm. — A1 N1

The lower bound is 5.25 cm. — A1 N1 [2]

(c) The percentage error

$= \left| \frac{(5.3 + 4.8 + 4.1 + 3.8) - 17.964}{17.964} \right| \times 100\%$ — (A1) for correct substitution

$= 0.200400801\%$

$= 0.200\%$ — A1 N2 [2]

3. (a) The volume

$= (15.75)(8.95)(7.15)$ (M1) for valid approach

$= 1007.881875\ \text{cm}^3$ A1 N2

[2]

(b) The upper bound is 7.5 cm. A1 N1

The lower bound is 6.5 cm. A1 N1

[2]

(c) The percentage error

$= \left| \dfrac{(16)(9)(7) - 1007.881875}{1007.881875} \right| \times 100\%$ (A1) for correct substitution

$= 0.011720123\%$

$= 0.0117\%$ A1 N2

[2]

4. (a) $L = \sqrt{39.063125^2 - 10.937675^2}$ (M1) for valid approach

$L = 37.5006\ \text{km}$ A1 N2

[2]

(b) $37.495\ \text{km} \le L < 37.505\ \text{km}$ A2 N2

[2]

(c) The percentage error

$= \left| \dfrac{\dfrac{(37.50)(10.94)}{2} - \dfrac{(37.5006)(10.937675)}{2}}{\dfrac{(37.5006)(10.937675)}{2}} \right| \times 100\%$ (A1) for correct substitution

$= 0.019656488\%$

$= 0.0197\%$ A1 N2

[2]

Chapter 3 Solution

Exercise 3

1. (a) (i) $f(2)=3$ A1 N1

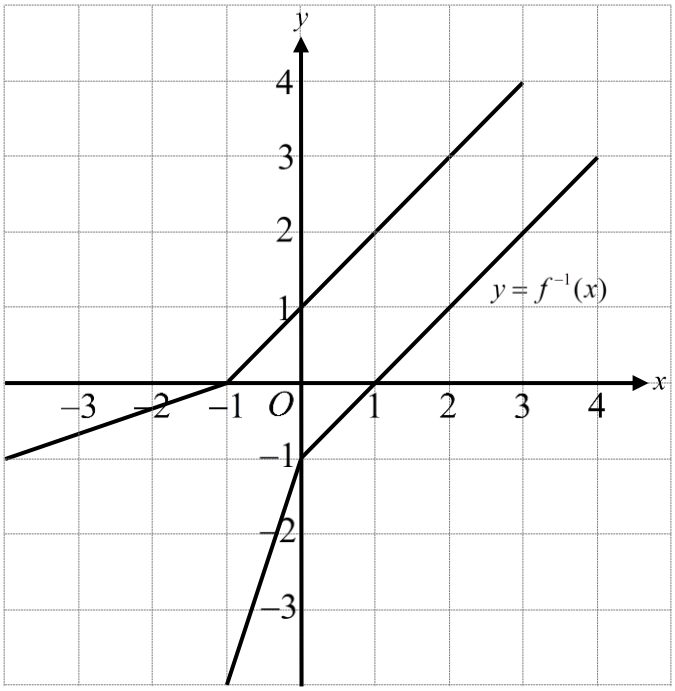
(ii) $f^{-1}(-1)=-4$ A2 N2

[3]

(b) For any two correct points from $(-1,-4)$, $(0,-1)$ or $(4,3)$ M1

For correct graph A2 N3

[3]



2. (a) (i) $f(-4) = 3$ A1 N1

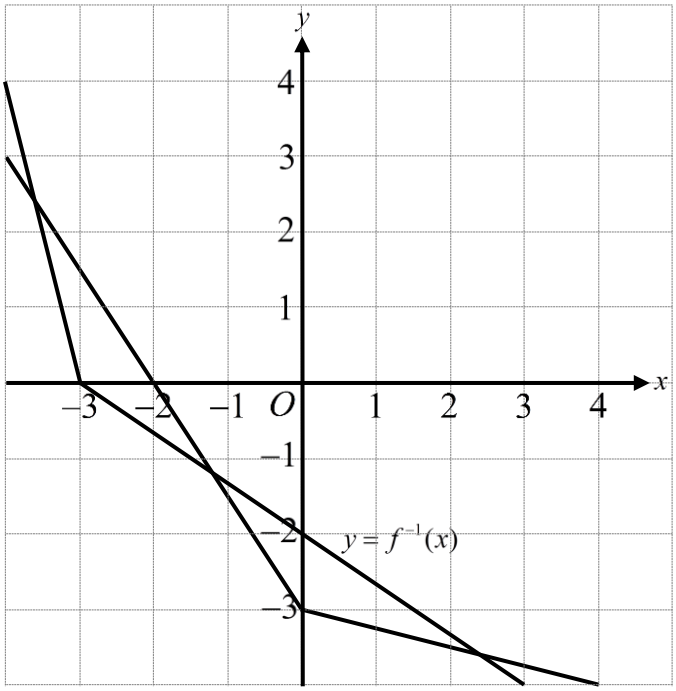
(ii) $f^{-1}(-4) = 4$ A2 N2

[3]

(b) For any two correct points from $(-4, 4)$, $(-3, 0)$ or $(3, -4)$ M1

For correct graph A2 N3

[3]



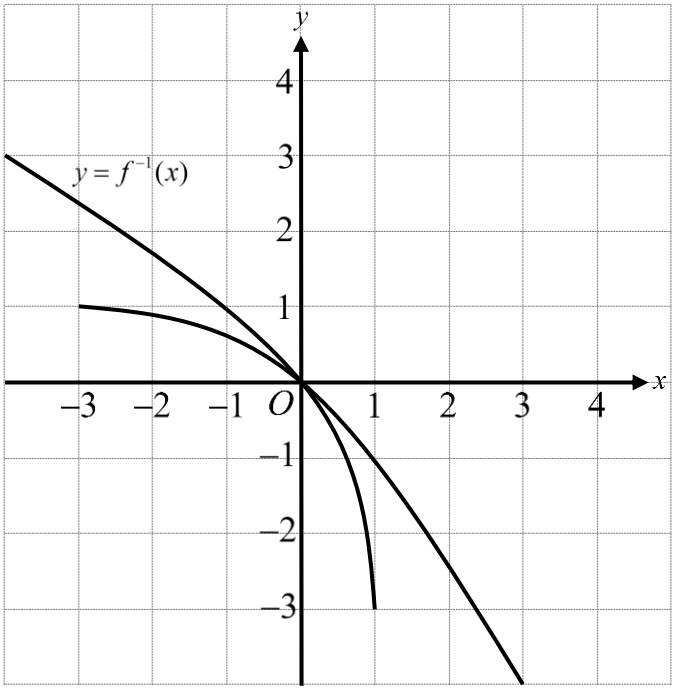
(c) When the displacement of the particle is -2, its velocity is 0. A1 N1

[1]

3. (a) For any two correct points from $(-4, 3)$, $(0, 0)$ or $(1, -3)$ M1

For correct graph A2 N3

[3]



(b) -1 A1 N1

[1]

(c) 0 A1 N1

[1]

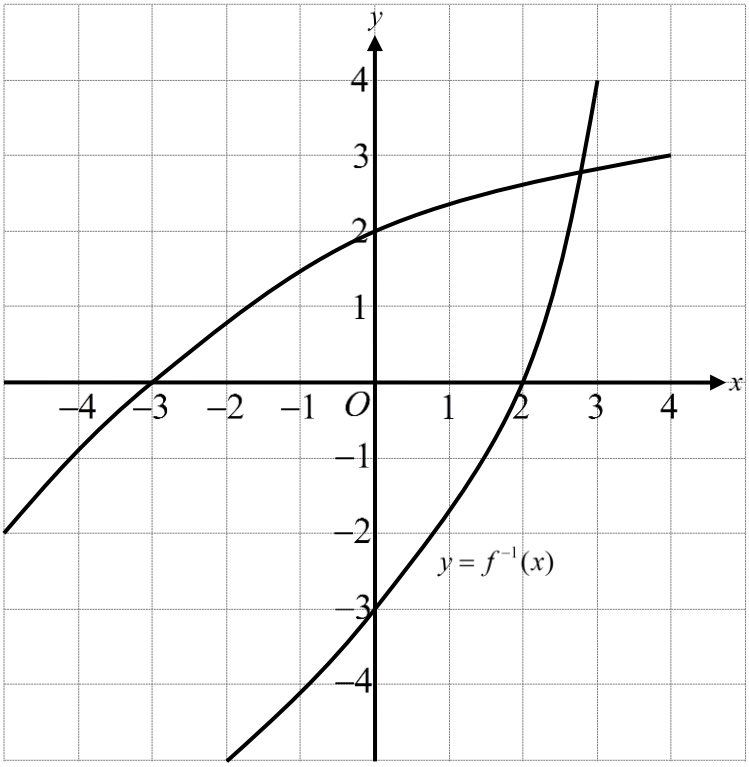
(d) -4 A1 N1

[1]

4. (a) For any two correct points from $(-2,-5)$, $(0,-3)$, $(2,0)$ or $(3,4)$ M1

For correct graph A2 N3

[3]



(b) -4 A1 N1

[1]

(c) 3 A1 N1

[1]

(d) When the displacement of the car is 0, its velocity is 2. A1 N1

[1]

Exercise 4

1. (a) $f(x)=0$

$\frac{2}{x+3}-5=0$ (M1) for setting equation

$\frac{2}{x+3}=5$

$x+3=\frac{2}{5}$

$x=-\frac{13}{5}$ A1 N2

[2]

(b) $x=-3$ A2 N2

[2]

(c) $y=-5$ A2 N2

[2]

2. (a) The y-intercept

$=f(0)$ (M1) for substitution

$=3-\frac{6}{1-0}$

$=-3$ A1 N2

[2]

(b) $x=1$ A2 N2

[2]

(c) $y=3$ A2 N2

[2]

3. (a) $x = 5$ A2 N2

[2]

(b) $y = 1$ A2 N2

[2]

(c) $\{x : x \neq 5\}$ A1 N1

[1]

(d) $f(x) > g(x)$

$\dfrac{x+2}{x-5} > x$

$\dfrac{x+2}{x-5} - x > 0$ M1

By considering the graph of $y = \dfrac{x+2}{x-5} - x$,

$x < -0.316625$ or $5 < x < 6.3166248$.

$\therefore x < -0.317$ or $5 < x < 6.32$ A1 N2

[2]

4. (a) $x = \dfrac{5}{2}$ A2 N2

[2]

(b) $y = \dfrac{7}{2}$ A2 N2

[2]

(c) $\left\{y : y \neq \dfrac{7}{2}\right\}$ A1 N1

[1]

(d) $f(x) \leq g(x)$

$\dfrac{7x-1}{2x-5} \leq x - 2$

$\dfrac{7x-1}{2x-5} - x + 2 \leq 0$ M1

By considering the graph of $y = \dfrac{7x-1}{2x-5} - x + 2$,

$0.7596297 \leq x < \dfrac{5}{2}$ or $x \geq 7.2403703$.

$\therefore 0.760 \leq x < \dfrac{5}{2}$ or $x \geq 7.24$ A1 N2

[2]

Exercise 5

1. (a) m represents the rate of change of the boiling point of water in degrees Celsius per 1 metre increase in vertical height above the sea level. A1 N1

[1]

(b) $64 = 100 + m(10000)$ (M1) for substitution

$-36 = 10000m$

$m = -0.0036$ A1 N2

[2]

(c) $84 = 100 - 0.0036x$ (M1) for substitution

$-16 = -0.0036x$

$x = 4444.444444$

Therefore, the height is 4440 m. A1 N2

[2]

2. (a) b represents the initial number of hotels. A1 N1

[1]

(b) $b = 143$ A1 N1

$193 = a(5) + 143$ M1

$50 = 5a$

$a = 10$ A1 N2

[3]

(c) The number of hotels

$= 10(8) + 143$ (M1) for substitution

$= 223$ A1 N2

[2]

3. (a) a represents the rate of change of the daily salary in dollars per 1 hour increase in working time. A1 N1

[1]

(b) b represents the fixed daily salary. A1 N1

[1]

(c) $b = 200$ A1 N1

$600 = a(8) + 200$ M1

$400 = 8a$

$a = 50$ A1 N2

[3]

(d) The daily salary

$= 50(0.5) + 200$ (M1) for substitution

$= \$225$ A1 N2

[2]

4. (a) p represents the rate of change of the area of the aluminium lamina in mm^2 per 1 degree Celsius increase in temperature. A1 N1

[1]

(b) q represents the area of the aluminium lamina at 0°C. A1 N1

[1]

(c) $q = 5$ A1 N1

$8 = p(60) + 5$ M1

$3 = 60p$

$p = 0.05$ A1 N2

[3]

(d) The difference of the areas

$= 0.05(140)$ (M1) for substitution

$= 7\ \text{mm}^2$ A1 N2

[2]

Chapter 4 Solution

Exercise 6

1. (a) $f(x)=0$ (M1) for setting equation

$x^2-6x+8=0$

$(x-2)(x-4)=0$ A1

$x=2$ or $x=4$

Hence, the x-intercepts are 2 and 4 respectively. A2 N4

[4]

(b) (i) $x=3$ A1 N1

(ii) The y-coordinate of the vertex

$=3^2-6(3)+8$ (M1) for substitution

$=-1$ A1 N2

[3]

2. (a) $f(x)=0$ (M1) for setting equation

$x^2-11x+10=0$

$(x-10)(x-1)=0$ A1

$x=10$ or $x=1$

Hence, the x-intercepts are 1 and 10 respectively. A2 N4

[4]

(b) (i) $x=5.5$ A1 N1

(ii) The y-coordinate of the vertex

$=5.5^2-11(5.5)+10$ (M1) for substitution

$=-20.25$ A1 N2

[3]

3. (a) $f(x) = 0$ (M1) for setting equation

$-2x^2 - 14x = 0$

$-2x(x+7) = 0$ A1

$x = 0$ or $x = -7$

Hence, the x-intercepts are 0 and -7 respectively. A2 N4

[4]

(b) (i) $x = -3.5$ A1 N1

(ii) The y-coordinate of the vertex

$= -2(-3.5)^2 - 14(-3.5)$ (M1) for substitution

$= 24.5$ A1 N2

[3]

4. (a) $f(x) = 0$ (M1) for setting equation

$13.5 - 1.5x^2 = 0$

$1.5(9 - x^2) = 0$ A1

$1.5(3+x)(3-x) = 0$

$x = -3$ or $x = 3$

Hence, the x-intercepts are -3 and 3 respectively. A2 N4

[4]

(b) (i) $x = 0$ A1 N1

(ii) The y-coordinate of the vertex

$= 13.5 - 1.5(0)^2$ (M1) for substitution

$= 13.5$ A1 N2

[3]

Exercise 7

1. (a) $f(x) = (x-7)(x+5)$ A2 N2 [2]

(b) $x = -5$ and $x = 7$ A2 N2 [2]

(c) $\{y : y \geq -36\}$ A1 N1 [1]

2. (a) $f(x) = -2(x+1)(x+6)$ A2 N2 [2]

(b) $x = -1$ and $x = -6$ A2 N2 [2]

(c) $\{y : y \leq 12.5\}$ A1 N1 [1]

3. (a) $p = 5$ and $q = 11$ A2 N2 [2]

(b) $-7.5 = a(10-5)(10-11)$ M1A1

$-7.5 = -5a$

$a = 1.5$ A1 N3 [3]

(c) $\{y : y \geq -13.5\}$ A1 N1 [1]

4. (a) $p = 0$ and $q = 18$ A2 N2 [2]

(b) $30 = a(0-15)(15-18)$ M1A1

$30 = 45a$

$a = \frac{2}{3}$ A1 N3 [3]

(c) $\{y : y \leq 54\}$ A1 N1 [1]

Exercise 8

1. (a) $x = 2$ A2 N2 [2]

(b) $(2, 17)$ A1 N1 [1]

(c) $\{y : y \leq 17\}$ A2 N2 [2]

2. (a) 4 A2 N2 [2]

(b) $(7, -2)$ A1 N1 [1]

(c) $\{y : y \geq -2\}$ A2 N2 [2]

3. (a) $(-5, 12.5)$ A1 N1 [1]

(b) Note that the another x-intercept is $-r$. (A1) for correct value

$\frac{-r+0}{2} = -5$ (M1) for valid approach

$-r = -10$

$r = 10$ A1 N3

$12.5 = a(-5)(-5+10)$ (M1) for substitution

$12.5 = -25a$

$a = -0.5$ A1 N2 [5]

4. (a) -5 A1 N1 [1]

(b) Note that the x-intercept are $-r$ and -1.

$\frac{-r+(-1)}{2} = -2.5$ (M1) for valid approach

$-r - 1 = -5$

$-r = -4$

$r = 4$ A1 N2

$-4 = a(0+4)(0+1)$ (M1) for substitution

$-4 = 4a$

$a = -1$ A1 N2 [4]

(c) t A1 N1 [1]

Exercise 9

1. (a) The y-intercept

$= -(0-100)^2 + 80$ (M1) for substitution

$= -9920$ A1 N2

[2]

(b) $-(x-100)^2 + 80 \geq 16$

$-(x-100)^2 + 64 \geq 0$ (M1) for setting inequality

By considering the graph of $y = -(x-100)^2 + 64$,

$92 \leq x \leq 108$.

$\therefore p = 108$ A1 N2

[2]

(c) 100 A1 N1

[1]

2. (a) 5400 A1 N1

[1]

(b) $5064 \leq x^2 - 40x + 5400 \leq 5400$

$5064 \leq x^2 - 40x + 5400$ and $x^2 - 40x + 5400 \leq 5400$ (M1) for valid approach

$x^2 - 40x + 336 \geq 0$ and $x^2 - 40x \leq 0$

By considering the graphs of $y = x^2 - 40x + 336$

and $y = x^2 - 40x$, $28 \leq x \leq 40$.

$\therefore p = 28$, $q = 40$ A2 N3

[3]

(c) $(20, 5000)$ A2 N2

[2]

3. (a) The required cost

$= 0.5(200-60)^2 + 40$ (M1) for substitution

$= 9840$ A1 N2

[2]

(b) $0.5(x-60)^2 + 40 \leq 240$

$0.5(x-60)^2 - 200 \leq 0$ (M1) for setting inequality

By considering the graph of $y = 0.5(x-60)^2 - 200$,

$40 \leq x \leq 80$. A1 N2

[2]

(c) 40 A1 N1

[1]

(d) 60 A1 N1

[1]

4. (a) The average profit

$= -0.25(22-20)^2 + 21$ (M1) for substitution

$= 20$ A1 N2

[2]

(b) $-0.25(x-20)^2 + 21 \geq 17$

$-0.25(x-20)^2 + 4 \geq 0$ (M1) for setting inequality

By considering the graph of $y = -0.25(x-20)^2 + 4$,

$16 \leq x \leq 24$. A1 N2

[2]

(c) $x \leq 18$

By considering the graph of $y = -0.25(x-20)^2 + 21$,

$y \leq 20$. (M1) for valid approach

Therefore, the maximum average profit is 20. A1 N2

[2]

Exercise 10

1. (a) $A=(30-2x)(20-2x)$ (M1) for valid approach

$A=600-60x-40x+4x^2$

$A=600-100x+4x^2$ A1 N2

[2]

(b) $A=299$

$600-100x+4x^2=299$ (M1) for setting equation

$4x^2-100x+301=0$ (A1) for correct equation

$(2x-43)(2x-7)=0$

$2x-43=0$ or $2x-7=0$

$x=21.5$ (*Rejected*) or $x=3.5$ A1 N3

[3]

(c) $1046.5\,\text{cm}^3$ A1 N1

[1]

2. (a) $x^2=(x-1)^2+(x-18)^2$ (M1) for valid approach

$x^2=x^2-2x+1+x^2-36x+324$

$x^2-38x+325=0$ A1 N2

[2]

(b) $x^2-38x+325=0$

$(x-13)(x-25)=0$ (M1) for factorization

$x-13=0$ or $x-25=0$

$x=13$ (*Rejected*) or $x=25$ A1 N2

[2]

(c) The painting cost per cm^2

$=\dfrac{1680}{\frac{1}{2}(25-18)(25-1)}$ M1

$=\$20/\text{cm}^2$ A1 N2

[2]

3. (a) $Q \geq 7.5$

$75-15r \geq 7.5$ (M1) for setting inequality

$67.5 \geq 15r$

$r \leq 4.5$

Therefore, the maximum cost is \$4.5. A1 N2

[2]

(b) $P = (75-15r)(r+2)$ (M1) for valid approach

$P = 75r + 150 - 15r^2 - 30r$

$P = -15r^2 + 45r + 150$ A1 N2

[2]

(c) $P = -15r^2 + 45r + 150$

By considering the graph of $y = -15r^2 + 45r + 150$, the maximum value of P is 183.75. (M1) for valid approach

Therefore, the maximum weekly profit is \$183.75. A1 N2

[2]

4. (a) By considering the graph of $y = -0.5x^2 + 2x + 10$, the maximum value of y is 12. (M1) for valid approach

Therefore, the maximum height is 12 m. A1 N2

[2]

(b) 4 m A1 N1

[1]

(c) $-0.5x^2 + 2x + 10 = 0$ (M1) for setting equation

By considering the graph of $y = -0.5x^2 + 2x + 10$, the x-intercept is 6.8989795. (M1) for valid approach

Therefore, the horizontal distance is 6.90 m. A1 N3

[3]

Chapter 5 Solution

Exercise 11

1. (a) $9 = a - b^{-0}$ (M1) for substitution

$9 = a - 1$

$a = 10$ A1 N2

[2]

(b) $\frac{262}{27} = 10 - b^{-3}$ (M1) for substitution

$-\frac{8}{27} + b^{-3} = 0$

By considering the graph of $y = -\frac{8}{27} + b^{-3}$,

$b = 1.5$. A1 N2

[2]

(c) $y = 10$ A2 N2

[2]

2. (a) $11 = p \times q^0 + 7$ (M1) for substitution

$11 = p + 7$

$p = 4$ A1 N2

[2]

(b) $8 = 4 \times q^2 + 7$ (M1) for substitution

$1 = 4q^2$

$q^2 = \frac{1}{4}$

$q = -\sqrt{\frac{1}{4}}$ or $q = \sqrt{\frac{1}{4}}$

$q = -\frac{1}{2}$ (*Rejected*) or $q = \frac{1}{2}$ A1 N2

[2]

(c) $\{y : y > 7\}$ A2 N2

[2]

3. (a) $2 = 3 \times 2^{-p(0)} + q$ (M1) for substitution

$2 = 3 + q$

$q = -1$ A1 N2

$47 = 3 \times 2^{-p(-2)} - 1$ (M1) for substitution

$48 - 3 \times 2^{2p} = 0$

By considering the graph of $y = 48 - 3 \times 2^{2p}$,

$p = 2$. A1 N2

[4]

(b) $0 = 3 \times 2^{-2x} - 1$ (M1) for substitution

By considering the graph of $y = 3 \times 2^{-2x} - 1$,

$x = 0.7924813$.

Thus, the x-intercept is $x = 0.792$. A1 N2

[2]

(c) 0 A1 N1

[1]

4. (a) $6 = 2 \times a^0 + b$ (M1) for substitution

$6 = 2 + b$

$b = 4$ A1 N2

$166 = 2 \times a^4 + 4$ (M1) for substitution

$162 - 2 \times a^4 = 0$

By considering the graph of $y = 162 - 2 \times a^4$,

$a = 3$. A1 N2

[4]

(b) Increases A1 N1

[1]

(c) 1 A1 N1

[1]

Exercise 12

1. (a) $18000 A1 N1

[1]

(b) The price of the car

$= 2000 + 16000e^{-\frac{6}{8}}$ (M1) for substitution

$= 9557.864844$

$= \$9560$ A1 N2

[2]

(c) $2000 + 16000e^{-\frac{t}{8}} < 7000$ (M1) for valid approach

$16000e^{-\frac{t}{8}} - 5000 < 0$

By considering the graph of $y = 16000e^{-\frac{t}{8}} - 5000$,

$t = 9.3052065$.

Thus, the time taken is 9.31 years. A1 N2

[2]

(d) $2000 A1 N1

[1]

2. (a) 270 A1 N1

[1]

(b) The increase in the number of bacteria

$= \left(115 + 155e^{\frac{7}{25}}\right) - 270$ (M1) for substitution

$= 50.08512091$

$= 50$ A1 N2

[2]

(c) $1200 = 115 + 155e^{\frac{t}{25}}$ (M1) for valid approach

$1085 - 155e^{\frac{t}{25}} = 0$

By considering the graph of $y = 1085 - 155e^{\frac{t}{25}}$,

$t = 48.647754$.

Thus, the time taken is 48.6 days. A1 N2

[2]

3. (a) The initial price of the computer system. A1 N1

[1]

(b) $840 = 90 + A \times 0.7^0$ (M1) for substitution

$840 = 90 + A$

$A = 750$ A1 N2

[2]

(c) $420 = 90 + 750 \times 0.7^t$ (M1) for valid approach

$330 - 750 \times 0.7^t = 0$

By considering the graph of $y = 330 - 750 \times 0.7^t$,

$t = 2.3017612$.

Thus, the time required is 2.30 years. A1 N2

[2]

(d) EUR 90 A1 N1

[1]

4. (a) The initial amount of electric charge stored. A1 N1

[1]

(b) $4 = p - q^0$ (M1) for substitution

$4 = p - 1$

$p = 5$ A1 N2

$4.488 = 5 - q^3$ (M1) for valid approach

$0.512 - q^3 = 0$

By considering the graph of $y = 0.512 - q^3$,

$q = 0.8$. A1 N2

[4]

(c) $4.3 = 5 - 0.8^t$ (M1) for valid approach

$0.7 - 0.8^t = 0$

By considering the graph of $y = 0.7 - 0.8^t$,

$t = 1.5984103$.

Thus, the time required to reach the charge is 1.60 hours. A1 N2

[2]

Exercise 13

1. (a) The required magnitude

$= 3\log_{10}(2\times 5\times 10^3)$ (M1) for correct formula

$= 12$ A1 N2

[2]

(b) $0.9 = 3\log_{10}(2E)$ (M1) for substitution

$0.3 = \log_{10}(2E)$

$\log_{10}(2E) - 0.3 = 0$

By considering the graph of $y = \log_{10}(2E) - 0.3$,

$E = 0.997631157$.

Thus, the amount of energy released is 0.998 units. A1 N2

[2]

2. $139.8 = 120 + 9.9\log_{10} I$ (M1) for substitution

$9.9\log_{10} I - 19.8 = 0$

By considering the graph of $y = 9.9\log_{10} I - 19.8$, $I = 100$. (A1) for correct working

$169.5 = 120 + 9.9\log_{10} I$ (M1) for substitution

$9.9\log_{10} I - 49.5 = 0$

By considering the graph of $y = 9.9\log_{10} I - 49.5$,

$I = 100000$. (A1) for correct working

The required ratio

$= 100000 : 100$

$= 1000 : 1$ A1 N5

[5]

3. (a) The y-intercept

$= 2(0) + 3\log_{10}(0+2)$ (M1) for substitution

$= 0.903089987$

$= 0.903$ A1 N2

[2]

(b) 1 A1 N1

[1]

(c) $2x + 3\log_{10}(x+2) = x^2$ (M1) for substitution

$2x + 3\log_{10}(x+2) - x^2 = 0$

By considering the graph of

$y = 2x + 3\log_{10}(x+2) - x^2$,

$x = -0.3004$ or $x = 2.7399173$.

Thus, $x = -0.300$ or $x = 2.74$. A2 N3

[3]

4. $f(0) = 0 + \log_{10}(0+10) + 2$ (M1) for substitution

$f(0) = 3$

Thus, the coordinates of B are $(0, 3)$. (A1) for correct working

$f(x) = 0$ (M1) for substitution

$x + \log_{10}(x+10) + 2 = 0$

By considering the graph of $y = x + \log_{10}(x+10) + 2$,

$x = -2.854059$.

Thus, the coordinates of A are $(-2.854059, 0)$. (A1) for correct working

The required area

$= \dfrac{(2.854059)(3)}{2}$ (M1) for valid approach

$= 4.2810885$

$= 4.28$ A1 N6

[6]

Chapter 6 Solution

Exercise 14

1. (a) $a+b=10$ A1 N1

$9a+3b=42$ A1 N1

(or $36a+6b=120$)

[2]

(b) $a=2,\ b=8$ A2 N2

[2]

(c) $h=-\dfrac{8}{2(2)}$ (M1) for substitution

$h=-2$

Thus, the equation of the axis of symmetry is

$x=-2$. A1 N2

[2]

2. (a) $a+b=1$ (A1) for correct substitution

$8a+b=15$ (A1) for correct substitution

Solving, we have $a=2$ and $b=-1$. A2 N4

[4]

(b) $2c^3-1=-129$ (M1) for substitution

$2c^3=-128$

$c^3=-64$

$c=-4$ A1 N2

[2]

3. (a) $\dfrac{a}{3-b}=1$ (A1) for correct substitution

$a=3-b$

$\dfrac{a}{3-4}=-2b$ (A1) for correct substitution

$a=2b$

Solving, we have $a=2$ and $b=1$. A2 N4

[4]

(b) 1, 4 A1 N1

[1]

(c) $y=0$ A1 N1

[1]

4. (a) $\dfrac{1}{\left(\frac{1}{4}\right)^2} = p+q$ (A1) for correct substitution

$p+q=16$

$\dfrac{1}{\left(\frac{1}{2}\right)^2} = p-q$ (A1) for correct substitution

$p-q=4$

Solving, we have $p=10$ and $q=6$. A2 N4

$r=\dfrac{1}{r^2}$ (A1) for correct substitution

$r^3=1$

$r=1$ A1 N2

[6]

(b) 1, 4, 16 A1 N1

[1]

(c) $x=0$ A1 N1

[1]

Exercise 15

1. (a) $x + y = 30$ A1 N1

[1]

(b) $3x + y = 82$ A1 N1

[1]

(c) $x = 26,\ y = 4$ A2 N2

[2]

(d) The total points
$= 3(15) + 15$ (M1) for substitution
$= 60$ A1 N2

[2]

2. (a) $a + 190b = 8.25$ A1 N1

[1]

(b) $a + 220b = 9.21$ A1 N1

[1]

(c) $a = 2.17,\ b = 0.032$ A2 N2

[2]

(d) $2.17 + 0.032T = 9.85$ (M1) for substitution
$0.032T = 7.68$
$T = 240$
Therefore, the temperature is $240\ ^{\circ}\text{C}$. A1 N2

[2]

3. (a) $18000 = 2p + q$ (A1) for correct substitution
$22000 = 7p + q$ (A1) for correct substitution
Solving, we have $p = 800$ and $q = 16400$. A2 N4

[4]

(b) p represents the increase of the number of flats per year. A1 N1

[1]

(c) q represents the initial number of flats. A1 N1

[1]

4. (a) Let x and y be the price of one CD and that of one DVD respectively.

$10x + 7y = 76.5$ (A1) for correct substitution

$8x + 11y = 90.9$ (A1) for correct substitution

Solving, we have $x = 3.8$ and $y = 5.5$.

Therefore, the price of one CD and that of one DVD are USD 3.8 and USD 5.5 respectively. A2 N4

[4]

(b) The amount of change

$= 100 - (7(3.8) + 10(5.5))$ (M1) for valid approach

$= \text{USD}\,18.4$ A1 N2

[2]

Exercise 16

1. (a) (i) $998 = a(1)^2 + b(1) + c$ A1

$a + b + c = 998$ AG N0

(ii) $982 = a(3)^2 + b(3) + c$ A1

$9a + 3b + c = 982$ AG N0

(iii) $36a + 6b + c = 928$ A1 N1

[3]

(b) $\begin{cases} a + b + c = 998 \\ 9a + 3b + c = 982 \\ 36a + 6b + c = 928 \end{cases}$ (M1) for valid approach

$a = -2$, $b = 0$ and $c = 1000$ A3 N4

[4]

2. (a) (i) $x + y + z = 8400$ A1 N1

(ii) $x + z = y - 6288$ A1

$x - y + z = -6288$ AG N0

[2]

(b) $42x + 84y + 21z = 655872$ A1 N1

[1]

(c) $\begin{cases} x + y + z = 8400 \\ x - y + z = -6288 \\ 42x + 84y + 21z = 655872 \end{cases}$ (M1) for valid approach

$x = 800$, $y = 7344$ and $z = 256$ A3 N4

[4]

3. (a) (i) $\begin{cases} 10a+12b+13c=150 \\ 14a+8b+19c=178 \\ 22a+23b+7c=230 \end{cases}$ A2 N2

(ii) $a=5$, $b=4$ and $c=4$ A3 N3

[5]

(b) The total price
$=5(30)+4(0)+4(35)$ (A1) for substitution
$=\$290$ A1 N2

[2]

4. (a) (i) $\begin{cases} 30x+16y=152 \\ 23x+15y+8z=114 \\ 11x+17y+18z=60 \end{cases}$ A2 N2

(ii) $x=4$, $y=2$ and $z=-1$ A3 N3

[5]

(b) A team drops 1 point for losing a game. A1 N1

[1]

Chapter 7 Solution

Exercise 17

1. (a) $d = \frac{u_5 - u_1}{5-1}$ (M1) for finding d

$d = \frac{-1-27}{4}$

$d = -7$ A1 N2

[2]

(b) $u_{25} = u_1 + (25-1)d$ (A1) for correct formula

$u_{25} = 27 + (25-1)(-7)$

$u_{25} = -141$ A1 N2

[2]

(c) $S_{25} = \frac{25}{2}\left[2u_1 + (25-1)d\right]$ (A1) for correct formula

$S_{25} = \frac{25}{2}\left[2(27) + (25-1)(-7)\right]$

$S_{25} = -1425$ A1 N2

[2]

2. (a) $d = \frac{u_7 - u_1}{7-1}$ (M1) for finding d

$d = \frac{6.5-3.5}{6}$

$d = 0.5$ A1 N2

[2]

(b) $u_{42} = u_1 + (42-1)d$ (A1) for correct formula

$u_{42} = 3.5 + (42-1)(0.5)$

$u_{42} = 24$ A1 N2

[2]

(c) $S_{84} = \frac{84}{2}\left[2u_1 + (84-1)d\right]$ (A1) for correct formula

$S_{84} = \frac{84}{2}\left[2(3.5) + (84-1)(0.5)\right]$

$S_{84} = 2037$ A1 N2

[2]

3. (a) $d = \frac{u_{10} - u_2}{10 - 2}$ (M1) for finding d

$d = \frac{24 - 0}{8}$

$d = 3$ A1 N2

[2]

(b) $u_4 = u_2 + 2d$ (A1) for correct formula

$u_4 = 0 + (2)(3)$

$u_4 = 6$ A1 N2

[2]

(c) $S_{10} = \frac{10}{2}\left[2u_1 + (10-1)d\right]$ (A1) for correct formula

$S_{10} = \frac{10}{2}\left[2(-3) + (10-1)(3)\right]$

$S_{10} = 105$ A1 N2

[2]

4. (a) $d = \frac{u_8 - u_3}{8 - 3}$ (M1) for finding d

$d = \frac{-\frac{22}{3} - \left(-\frac{2}{3}\right)}{5}$

$d = -\frac{4}{3}$ A1 N2

[2]

(b) $u_{11} = u_8 + 3d$ (A1) for correct formula

$u_{11} = -\frac{22}{3} + 3\left(-\frac{4}{3}\right)$

$u_{11} = -\frac{34}{3}$ A1 N2

[2]

(c) $S_{40} = \frac{40}{2}\left[2u_1 + (40-1)d\right]$ (A1) for correct formula

$S_{40} = \frac{40}{2}\left[2(2) + (40-1)\left(-\frac{4}{3}\right)\right]$ (A1) for substitution

$S_{40} = -960$ A1 N3

[3]

Exercise 18

1. (a) $d = 1.9 - 1.5$

$d = 0.4$ (A1) for correct value

The required length

$= 1.5 + (20 - 1)(0.4)$ (M1) for substitution

$= 9.1\,\text{m}$ A1 N3

[3]

(b) The perimeter

$= S_{20}$ (M1) for valid approach

$= \frac{20}{2}\left[2(1.5) + (20 - 1)(0.4)\right]$ (A1) for substitution

$= 106\,\text{m}$ A1 N3

[3]

2. (a) $d = 23 - 12$

$d = 11$ (A1) for correct value

$221 = 12 + (n - 1)(11)$ (M1) for substitution

$209 = 11(n - 1)$

$19 = n - 1$

$n = 20$ A1 N3

[3]

(b) The total number

$= u_{18} + u_{19} + u_{20}$ (M1) for valid approach

$= (221 - 22) + (221 - 11) + 221$ (A1) for substitution

$= 630$ A1 N3

[3]

3. (a) $d = 9300 - 7500$ (M1) for finding d

$d = 1800$

The required cost

$= u_{10}$ (M1) for valid approach

$= 7500 + (10-1)(1800)$

$= \$23700$ A1 N3

[3]

(b) $S_n < 340000$ (M1) for setting inequality

$\frac{n}{2}\left[2(7500) + (n-1)(1800)\right] < 340000$ (A1) for substitution

$900n^2 + 6600n - 340000 < 0$

By considering the graph of

$y = 900n^2 + 6600n - 340000$, $n < 16.112672$.

$\therefore n = 16$ (A1) for correct value

Thus, the greatest possible length is 160 m. A1 N4

[4]

4. (a) $d = 32 - 30$ (M1) for finding d

$d = 2$

The required number

$= u_{24}$ (M1) for valid approach

$= (30-2) + (24-1)(2)$

$= 74$ A1 N3

[3]

(b) The total number of seats in the theatre

$= S_{24}$ (M1) for valid approach

$= \frac{24}{2}\left[2(30-2) + (24-1)(2)\right]$

$= 1224$ (A1) for correct value

The total income

$= (1224)(75)$ (M1) for valid approach

$= \$91800$ A1 N4

[4]

Exercise 19

1. (a) (i) $u_4 = 43$

$u_1 + (4-1)d = 43$ (M1) for valid approach

$u_1 + 3d = 43$ A1 N2

(ii) $S_{80} = -5320$

$\frac{80}{2}\left[2u_1 + (80-1)d\right] = -5320$ (M1) for valid approach

$80u_1 + 3160d = -5320$ A1 N2

(iii) $u_1 = 52$ A1 N1

$d = -3$ A1 N1

[6]

(b) $u_n = 52 + (n-1)(-3)$ (M1) for valid approach

$u_n = -3n + 55$ A1 N2

[2]

(c) $u_n > 0$

$-3n + 55 > 0$ (M1) for setting inequality

$-3n > -55$

$n < \frac{55}{3}$ (A1) for correct value

Thus, the greatest value of n is 18. A1 N3

[3]

(d) $S_n = \frac{n}{2}\left[2(52) + (n-1)(-3)\right]$ (M1) for valid approach

$S_n = \frac{n}{2}(-3n + 107)$

$S_n = -\frac{3}{2}n^2 + \frac{107}{2}n$ A1 N2

[2]

(e) $S_n = -4425$

$-\frac{3}{2}n^2 + \frac{107}{2}n = -4425$ (M1) for setting equation

$3n^2 - 107n - 8850 = 0$ (M1) for quadratic equation

$(3n + 118)(n - 75) = 0$

$n = -\frac{118}{3}$ (*Rejected*) or $n = 75$ A1 N3

[3]

2. (a) (i) $u_{11} = 17$

$u_1 + (11-1)d = 17$ (M1) for valid approach

$u_1 + 10d = 17$ A1 N2

(ii) $S_{96} = 4512$

$\frac{96}{2}\left[2u_1 + (96-1)d\right] = 4512$ (M1) for valid approach

$96u_1 + 4560d = 4512$ A1 N2

(iii) $u_1 = 9$ A1 N1

$d = 0.8$ A1 N1

[6]

(b) $u_n = 9 + (n-1)(0.8)$ (M1) for valid approach

$u_n = 0.8n + 8.2$ A1 N2

[2]

(c) $u_n < 147$

$0.8n + 8.2 < 147$ (M1) for setting inequality

$0.8n < 138.8$

$n < 173.5$ (A1) for correct value

Thus, the greatest value of n is 173. A1 N3

[3]

(d) $u_{49} + u_{50} + u_{51} + \cdots + u_{95} + u_{96}$

$= S_{96} - S_{48}$ (M1) for valid approach

$= 4512 - \frac{48}{2}\left[2(9) + (48-1)(0.8)\right]$ (A1) for substitution

$= 3177.6$ A1 N3

[3]

(e) $S_n = 4949$

$\frac{n}{2}\left[2(9) + (n-1)(0.8)\right] = 4949$ (M1) for setting equation

$\frac{n}{2}\left[0.8n + 17.2\right] = 4949$

$0.4n^2 + 8.6n - 4949 = 0$

$2n^2 + 43n - 24745 = 0$ (M1) for quadratic equation

$(2n + 245)(n - 101) = 0$

$n = -\frac{245}{2}$ (*Rejected*) or $n = 101$ A1 N3

[3]

3. (a) (i) $u_1 = 60,\ u_2 = 57$ A2 N2

(ii) -3 A1 N1

[3]

(b) $u_n \le -21$

$63 - 3n \le -21$ (M1) for setting inequality

$-3n \le -84$

$n \ge 28$ (A1) for correct value

Thus, the smallest value of n is 28. A1 N3

[3]

(c) (i) 60 A1 N1

(ii) $S_n = \frac{n}{2}\left[2(60) + (n-1)(-3)\right]$ (M1) for valid approach

$S_n = \frac{n}{2}\left[-3n + 123\right]$

$S_n = -\frac{3}{2}n^2 + \frac{123}{2}n$ A1 N2

[3]

(d) $S_n = 0$

$-\frac{3}{2}n^2 + \frac{123}{2}n = 0$ (M1) for setting inequality

$n^2 - 41n = 0$

$n(n-41) = 0$ (A1) for factorization

$n = 0$ (*Rejected*) or $n = 41$ A1 N3

[3]

(e) $\sum_{r=11}^{20} u_r$

$= S_{20} - S_{10}$ (M1) for valid approach

$= \left[-\frac{3}{2}(20)^2 + \frac{123}{2}(20)\right] - \left[-\frac{3}{2}(10)^2 + \frac{123}{2}(10)\right]$ (A1) for substitution

$= 165$ A1 N3

[3]

4. (a) (i) $S_1 = \frac{5}{7},\ S_2 = \frac{3}{2}$ A2 N2

(ii) $u_1 = \frac{5}{7},\ u_2 = \frac{11}{14}$ A2 N2

(iii) The common difference

$= \frac{11}{14} - \frac{5}{7}$ (A1) for substitution

$= \frac{1}{14}$ A1 N2

[6]

(b) $\sum_{r=1}^{n} u_r > 100$

$\frac{n}{2}\left[2\left(\frac{5}{7}\right) + (n-1)\left(\frac{1}{14}\right)\right] > 100$ (A1) for substitution

$\frac{1}{28}n^2 + \frac{19}{28}n - 100 > 0$

By considering the graph of

$y = \frac{1}{28}n^2 + \frac{19}{28}n - 100,\ n > 44.261045.$ (A1) for correct value

Thus, the smallest value of n is 45. A1 N3

[3]

(c) $\sum_{r=50}^{100} u_r$

$= S_{100} - S_{49}$ (M1) for valid approach

$= \left[\frac{100^2 + 19(100)}{28}\right] - \left[\frac{49^2 + 19(49)}{28}\right]$ (A1) for substitution

$= 306$ A1 N3

[3]

(d) $u_n = \frac{5}{7} + (n-1)\left(\frac{1}{14}\right)$ (M1) for valid approach

$u_n = \frac{1}{14}n + \frac{9}{14}$ A1 N2

[2]

(e) $u_n + 2u_{n+1} = \frac{58}{7}$

$u_n + 2\left(u_n + \frac{1}{14}\right) = \frac{58}{7}$ (M1) for valid approach

$3u_n = \frac{57}{7}$

$u_n = \frac{19}{7}$

$\frac{1}{14}n + \frac{9}{14} = \frac{19}{7}$ (A1) for substitution

$n = 29$ A1 N3

[3]

Chapter 8 Solution

Exercise 20

1. (a) $r = \frac{1}{4}$ A1 N1

[1]

(b) $u_8 = u_1 \times r^{8-1}$

$u_8 = 1024 \times \left(\frac{1}{4}\right)^{8-1}$ (A1) for substitution

$u_8 = \frac{1}{16}$ A1 N2

[2]

(c) $S_{12} = \frac{u_1(1-r^{12})}{1-r}$ (M1) for valid approach

$S_{12} = \frac{1024\left(1-\left(\frac{1}{4}\right)^{12}\right)}{1-\frac{1}{4}}$ (A1) for substitution

$S_{12} = 1365.333252$

$S_{12} = 1365$ A1 N3

[3]

2. (a) $r = \frac{4}{3}$ A1 N1

[1]

(b) $\sum_{n=1}^{7} u_n = \frac{u_1(1-r^7)}{1-r}$ (M1) for valid approach

$\sum_{n=1}^{7} u_n = \frac{576\left(1-\left(\frac{4}{3}\right)^7\right)}{1-\frac{4}{3}}$ (A1) for substitution

$\sum_{n=1}^{7} u_n = 11217.38272$

$\sum_{n=1}^{7} u_n = 11217$ A1 N3

[3]

(c) $243u_n = 1048576$

$243\left(576\times\left(\frac{4}{3}\right)^{n-1}\right) = 1048576$ (M1) for substitution

$243\left(576\times\left(\frac{4}{3}\right)^{n-1}\right) - 1048576 = 0$ (M1) for setting equation

By considering the graph of

$y = 243\left(576\times\left(\frac{4}{3}\right)^{n-1}\right) - 1048576,$

$n = 8.$ A1 N3

[3]

3. (a) $r = \dfrac{1.28}{1.024}$ (M1) for finding r

$r = 1.25$ A1 N2

[2]

(b) $u_n > 5$

$1.024 \times 1.25^{n-1} > 5$ (M1) for correct formula

$1.024 \times 1.25^{n-1} - 5 > 0$

By considering the graph of $y = 1.024 \times 1.25^{n-1} - 5$,

$n > 8.1062837$. (A1) for correct value

$\therefore$ The least value of n is 9. A1 N3

[3]

(c) $\sum_{n=1}^{10} u_n = S_{10}$

$\sum_{n=1}^{10} u_n = \dfrac{1.024(1-1.25^{10})}{1-1.25}$ (A1) for substitution

$\sum_{n=1}^{10} u_n = 34.05097266$

$\sum_{n=1}^{10} u_n = 34.1$ A1 N2

[2]

4. (a) $r = \dfrac{2.4}{1.5}$ (M1) for valid approach

$r = 1.6$ A1 N2

[2]

(b) $u_8 = u_1 \times r^{8-1}$

$u_8 = 1.5 \times 1.6^{8-1}$ (A1) for correct substitution

$u_8 = 40.2653184$

$u_8 = 40.27$ A1 N2

[2]

(c) $S_n < 100$

$\dfrac{1.5(1-1.6^n)}{1-1.6} < 100$ (M1) for correct formula

$-2.5(1-1.6^n) < 100$

$-2.5(1-1.6^n) - 100 < 0$

By considering the graph of

$y = -2.5(1-1.6^n) - 100$, $n < 7.9011562$. (A1) for correct value

Thus, the greatest value of n is 7. A1 N3

[3]

Exercise 21

1. (a) 1.331 m — A1 N1

[1]

(b) The length of the longest side

$= u_{10}$ — (M1) for valid approach

$= 1 \times 1.1^{10-1}$

$= 2.357947691$

$= 2.36$ m — A1 N2

[2]

(c) The perimeter

$= S_{10}$ — (M1) for valid approach

$= \frac{1(1-1.1^{10})}{1-1.1}$ — (A1) for substitution

$= 15.9374246$

$= 15.9$ m — A1 N3

[3]

2. (a) The required price

$= u_6$ — (M1) for valid approach

$= 100 \times 0.9^{6-1}$

$= 59.049$

$= \$59$ — A1 N2

[2]

(b) The required price

$= u_n$ — (M1) for valid approach

$= 100 \times 0.9^{n-1}$ — A1 N2

[2]

(c) The total income

$= 30S_{12}$ — (M1) for valid approach

$= 30\left(\frac{100(1-0.9^{12})}{1-0.9}\right)$ — (A1) for substitution

$= 21527.11391$

$= \$21527$ — A1 N3

[3]

3. (a) The total volume

$= S_{20}$ (M1) for valid approach

$= \dfrac{24000(1-0.95^{20})}{1-0.95}$ (A1) for substitution

$= 307926.7572$

$= 308000\ \text{cm}^3$ A1 N3

[3]

(b) $u_n < 10000$

$24000 \times 0.95^{n-1} < 10000$ (M1) for correct formula

$24000 \times 0.95^{n-1} - 10000 < 0$

By considering the graph of

$y = 24000 \times 0.95^{n-1} - 10000$, $n > 18.067898$. (A1) for correct value

$\therefore$ The number of wooden dolls is 2. A1 N3

[3]

4. (a) The distance travelled

$= u_5$ (M1) for valid approach

$= 120 \times 0.9^{5-1}$

$= 78.732$

$= 78.7\ \text{km}$ A1 N2

[2]

(b) The difference

$= u_5 - u_6$

$= u_5 - 0.9u_5$ (M1) for valid approach

$= 78.732 - 0.9(78.732)$

$= 7.8732$

$= 7.87\ \text{km}$ A1 N2

[2]

(c) $S_n = 1000$

$\dfrac{120(1-0.9^n)}{1-0.9} = 1000$ (M1) for correct formula

$1200(1-0.9^n) = 1000$

$1200(1-0.9^n) - 1000 = 0$

By considering the graph of

$y = 1200(1-0.9^n) - 1000$, $n = 17.005986$. (A1) for correct value

$\therefore$ The date is 17th February. A1 N3

[3]

Exercise 22

1. (a) The amount of insurance premium

$=1200+(4-1)(-15)$ (A1) for substitution

$=1155$ EUR A1 N2

[2]

(b) The exact value of the car

$=24000\times(1-15\%)^{6-1}$ (A1) for substitution

$=10648.9275$ EUR A1 N2

[2]

(c) $24000\times(1-15\%)^{n-1}<8000$ (M1) for setting inequality

$24000\times0.85^{n-1}-8000<0$ (A1) for correct inequality

By considering the graph of

$y=24000\times0.85^{n-1}-8000$, $n>7.7599036$.

Thus, the year is 2018. A1 N3

[3]

(d) $1200+(n-1)(-15)>24000\times(1-15\%)^{n-1}$ (M1) for setting inequality

$1200-15n+15>24000\times0.85^{n-1}$

$1215-15n-24000\times0.85^{n-1}>0$ (A1) for correct inequality

By considering the graph of

$y=1215-15n-24000\times0.85^{n-1}$, $n>21.22625$.

Thus, the year is 2032. A1 N3

[3]

(e) The total amount of insurance premium

$=\frac{21}{2}\left[2(1200)+(21-1)(-15)\right]$ M1A1

$=22050$ EUR A1 N3

[3]

2. (a) (i) v_n A1 N1

(ii) t_n A1 N1

(iii) u_n A1 N1

(iv) w_n A1 N1

[4]

(b) (i) $v_{100} = v_1 + (100-1)d$ (M1) for valid approach

$v_{100} = 50 + (100-1)(1000)$

$v_{100} = 99050$ A1 N2

(ii) The sum of the first 25 terms

$= \frac{25}{2}\left[2v_1 + (25-1)d\right]$ (M1) for valid approach

$= \frac{25}{2}\left[2(50) + (25-1)(1000)\right]$ (A1) for substitution

$= 301250$ A1 N3

[5]

(c) (i) t_7

$= t_1 \times r^{7-1}$ (M1) for valid approach

$= 50 \times 2^{7-1}$

$= 3200$ A1 N2

(ii) The sum of the first 14 terms

$= \frac{t_1(1-r^{14})}{1-r}$ (M1) for valid approach

$= \frac{50(1-2^{14})}{1-2}$ (A1) for substitution

$= 819150$ A1 N3

[5]

(d) $v_m \geq t_m$

$50 + (m-1)(1000) \geq 50 \times 2^{m-1}$ (M1) for setting inequality

$1000m - 950 \geq 50 \times 2^{m-1}$

$1000m - 950 - 50 \times 2^{m-1} \geq 0$ (A1) for correct inequality

By considering the graph of

$y = 1000m - 950 - 50 \times 2^{m-1},\ m \leq 8.1749071.$

Thus, $m = 8$. A1 N3

[3]

3. (a) Giselle's running distance

$= 2400 + (n-1)(200)$ (M1) for valid approach

$= (200n + 2200)$ m A1 N2

[2]

(b) $200x + 2200 > 10000$ (M1) for setting inequality

$200x > 7800$

$x > 39$ (A1) for correct value

Thus, $x = 40$. A1 N3

[3]

(c) The total running distance

$= \frac{16}{2}\left[2(2400) + (16-1)(200)\right]$ (M1) for valid approach

$= 62400$ m

$= 62.4$ km (A1) for correct unit

$= 6.24 \times 10^{1}$ km A1 N3

[3]

(d) Helena's running distance

$= 2000 \times (1+5\%)^{10-1}$ (M1) for valid approach

$= 3102.656432$

$= 3100$ m A1 N2

[2]

(e) $\frac{w}{2}\left[2(2400) + (w-1)(200)\right] < \frac{2000(1-1.05^{w})}{1-1.05}$ M1A1

$\frac{w}{2}(200w + 4600) < -40000(1-1.05^{w})$

$100w^{2} + 2300w + 40000(1-1.05^{w}) < 0$

By considering the graph of

$y = 100w^{2} + 2300w + 40000(1-1.05^{w})$,

$w > 42.551642$. (A1) for correct value

Thus, $w = 43$. A1 N4

[4]

4. (a) The coffee shop's profit

$= 2000 \times (1+15\%)^{9-1}$ (M1) for valid approach

$= 6118.045725$

$= 6120$ EUR A1 N2

[2]

(b) The coffee shop's total profit

$= \dfrac{2000(1-1.15^{10})}{1-1.15}$ (M1)(A1) for substitution

$= 40607.43648$

$= 40600$ EUR A1 N3

[3]

(c) The fast food shop's profit

$= 4000 + (16-1)(1100)$ (M1) for valid approach

$= 20500$ EUR A1 N2

[2]

(d) $4000 + (m-1)(1100) > 30000$ (M1) for setting inequality

$1100m + 2900 > 30000$

$1100m > 27100$

$m > 24.63636364$ (A1) for correct value

Thus, $m = 25$. A1 N3

[3]

(e) $\dfrac{n}{2}\left[2(4000) + (n-1)(1100)\right] < \dfrac{2000(1-1.15^{n})}{1-1.15}$ M1A1

$\dfrac{n}{2}(1100n + 6900) < -\dfrac{40000(1-1.15^{n})}{3}$

$550n^2 + 3450n + \dfrac{40000(1-1.15^{n})}{3} < 0$

By considering the graph of

$y = 550n^2 + 3450n + \dfrac{40000(1-1.15^{n})}{3}$,

$n > 25.142701$. (A1) for correct value

Thus, $n = 26$. A1 N4

[4]

Chapter 9 Solution

Exercise 23

1. (a) $P = 360000\left(1+\frac{3}{(100)(2)}\right)^{(2)(8)}$ (M1)(A1) for substitution

$P = 456834.7972$

$P = 456800 \text{ EUR}$ A1 N3

By TVM Solver :
N = 8
I% = 3
PV = –360000
PMT = 0
FV = ?
P / Y = 1
C / Y = 2
PMT : END

(M1)(A1) for correct values

$P = 456800 \text{ EUR}$ A1 N3

[3]

(b) $Q\left(1+\frac{3}{(100)(12)}\right)^{(12)(8)} = 456834.7972$ (M1)(A1) for correct equation

$Q(1.0025)^{96} = 456834.7972$

$Q = 359466.6239$

$Q = 359500 \text{ EUR}$ A1 N3

By TVM Solver :
N = 8
I% = 3
PV = ?
PMT = 0
FV = 456834.7972
P / Y = 1
C / Y = 12
PMT : END

(M1)(A1) for correct values

$Q = 359500 \text{ EUR}$ A1 N3

[3]

2. (a) The amount of money

$= 125000\left(1+\frac{8}{100}\right)^{12}$ (M1)(A1) for substitution

$= 314771.2646$

$= \$315000$ A1 N3

By TVM Solver :
N = 12
I% = 8
PV = –125000
PMT = 0
FV = ?
P / Y = 1
C / Y = 1
PMT : END

(M1)(A1) for correct values

The amount of money is \$315000. A1 N3

[3]

(b) $125000\left(1+\frac{8}{100}\right)^{t} = 250000$ (M1)(A1) for correct equation

$1.08^t = 2$

$1.08^t - 2 = 0$

By considering the graph of $y = 1.08^t - 2$,

$t = 9.0064683$.

Thus, the minimum number of years is 10. A1 N3

By TVM Solver :
N = ?
I% = 8
PV = –125000
PMT = 0
FV = 250000
P / Y = 1
C / Y = 1
PMT : END

(M1)(A1) for correct values

Thus, the minimum number of years is 10. A1 N3

[3]

3. (a) $P\left(1+\frac{4}{(100)(4)}\right)^{(4)(5)} = 87000$ (M1)(A1) for correct equation

$P(1.01)^{20} = 87000$

$P = 71300.36892$

$P = 71300$ A1 N3

By TVM Solver :
N = 5
I% = 4
PV = ?
PMT = 0
FV = 87000
P / Y = 1
C / Y = 4
PMT : END

(M1)(A1) for correct values

$P = 71300$ A1 N3

[3]

(b) $P\left(1+\frac{4}{(100)(4)}\right)^{4t} = 2.5P$ (M1)(A1) for correct equation

$1.01^{4t} = 2.5$

$1.01^{4t} - 2.5 = 0$

By considering the graph of $y = 1.01^{4t} - 2.5$,

$t = 23.021615$.

Thus, the minimum number of years is 24. A1 N3

By TVM Solver :
N = ?
I% = 4
PV = −71300
PMT = 0
FV = 178250
P / Y = 1
C / Y = 4
PMT : END

(M1)(A1) for correct values

Thus, the minimum number of years is 24. A1 N3

[3]

4. $640000\left(1+\frac{5}{(100)(2)}\right)^{2t_1}=1280000$ (M1)(A1) for correct equation

$1.025^{2t_1}=2$

$1.025^{2t_1}-2=0$

By considering the graph of $y=1.025^{2t_1}-2$,

$t_1=14.035517$.

By TVM Solver :
N = ?
I% = 5
PV = –640000
PMT = 0
FV = 1280000
P / Y = 1
C / Y = 2
PMT : END

(M1)(A1) for correct values

Thus, $t_1=14.035517$.

$640000\left(1+\frac{5}{(100)(4)}\right)^{4t_2}=1280000$ (M1)(A1) for correct equation

$1.0125^{4t_2}=2$

$1.0125^{4t_2}-2=0$

By considering the graph of $y=1.0125^{4t_2}-2$,

$t_2=13.949408$.

By TVM Solver :
N = ?
I% = 5
PV = –640000
PMT = 0
FV = 1280000
P / Y = 1
C / Y = 4
PMT : END

(M1)(A1) for correct values

Thus, $t_2=13.949408$.

$t_1-t_2=14.035517-13.949408$ (M1) for valid approach

$t_1-t_2=0.086109$

$t_1-t_2=0.0861$ A1 N6

[6]

Exercise 24

1. (a) $P = 54000\left(1+\frac{6}{(100)(12)}\right)^{(12)(10)}$ (M1)(A1) for substitution

$P = 98247.42364$

$P = 98000$ EUR A1 N3

By TVM Solver :
N = 10
I% = 6
PV = –54000
PMT = 0
FV = ?
P / Y = 1
C / Y = 12
PMT : END

(M1)(A1) for correct values

$P = 98000$ EUR A1 N3

[3]

(b) $54000\left(1+\frac{r}{(100)(4)}\right)^{(4)(10)} = 98247.42364$ (M1)(A1) for correct equation

$54000\left(1+\frac{r}{400}\right)^{40} - 98247.42364 = 0$

By considering the graph of

$y = 54000\left(1+\frac{r}{400}\right)^{40} - 98247.42364$, $r = 6.03005$.

Thus, $r = 6.03$. A1 N3

By TVM Solver :
N = 10
I% = ?
PV = –54000
PMT = 0
FV = 98247.42364
P / Y = 1
C / Y = 4
PMT : END

(M1)(A1) for correct values

Thus, $r = 6.03$. A1 N3

[3]

2. (a) $P\left(1+\frac{9}{(100)(2)}\right)^{(7)(2)}=1600000$ (M1)(A1) for substitution

$P=863956.5796$

$P=860000$ A1 N3

By TVM Solver :
N = 7
I% = 9
PV = ?
PMT = 0
FV = 1600000
P / Y = 1
C / Y = 2
PMT : END

(M1)(A1) for correct values

$P=860000$ A1 N3

[3]

(b) $863956.5796\left(1+\frac{9}{100}\right)^{n}=1600000$ (M1)(A1) for correct equation

$863956.5796(1.09)^{n}-1600000=0$

By considering the graph of

$y=863956.5796(1.09)^{n}-1600000$,

$n=7.1507643$.

Thus, $n=7.15$. A1 N3

By TVM Solver :
N = ?
I% = 9
PV = –863956.5796
PMT = 0
FV = 1600000
P / Y = 1
C / Y = 1
PMT : END

(M1)(A1) for correct values

Thus, $n=7.15$. A1 N3

[3]

3. $\left(1+\frac{12}{(100)(4)}\right)^{(4)(4)}=\left(1+\frac{12}{(100)(12)}\right)^{(12)(n)}$ (M1)(A1) for correct equation

$1.03^{16}=1.01^{12n}$ (M1) for simplification

$1.03^{16}-1.01^{12n}=0$

By considering the graph of $y=1.03^{16}-1.01^{12n}$,

$n=3.9608468$.

Thus, $n=3.96$. A1 N4

[4]

4. $\left(1+\frac{5}{(100)(2)}\right)^{(2)(8)}=\left(1+\frac{5}{100k}\right)^{(k)(7.98)}$ (M1)(A1) for correct equation

$1.025^{16}=\left(1+\frac{1}{20k}\right)^{7.98k}$ (M1) for simplification

$1.025^{16}-\left(1+\frac{1}{20k}\right)^{7.98k}=0$

By considering the graph of $y=1.025^{16}-\left(1+\frac{1}{20k}\right)^{7.98k}$,

$k=2.5125342$.

Thus, $k=2.51$. A1 N4

[4]

Exercise 25

1. (a) $P\left(1+\frac{7}{100}\right)^4 = 300000$ (M1)(A1) for substitution

$P = 228868.5636$

$P = 229000$ A1 N3

By TVM Solver :
N = 4
I% = 7
PV = ?
PMT = 0
FV = 300000
P / Y = 1
C / Y = 1
PMT : END

(M1)(A1) for correct values

$P = 229000$ A1 N3

[3]

(b) 5.4% A1 N1

[1]

(c) The real value of amount of money

$= 228868.5636\left(1+\frac{5.4}{100}\right)^4$ (A1) for substitution

$= 282454.558$

$= \$282000$ A1 N2

By TVM Solver :
N = 4
I% = 5.4
PV = –228868.5636
PMT = 0
FV = ?
P / Y = 1
C / Y = 1
PMT : END

(A1) for correct values

Thus, the real value is \$282000. A1 N2

[2]

2. (a) Let $r\%$ be the nominal annual interest rate compounded yearly.

$$(1+r\%)^9 = \left(1+\frac{12}{(100)(12)}\right)^{(12)(9)}$$ (M1)(A1) for substitution

$1+r\% = 1.01^{12}$

$r = 12.68250301$

The real interest rate per year

$= 12.68250301\% - 1.8\%$ (M1) for valid approach

$= 10.88250301\%$

$= 10.9\%$ A1 N4

[4]

(b) The real value of amount of interest

$$= 8500\left(1+\frac{10.88250301}{100}\right)^9 - 8500$$ (M1)(A1) for substitution

$= 13037.04494$

$= 13000 \text{ EUR}$ A1 N3

By TVM Solver :
N = 9
I% = 10.88250301
PV = –8500
PMT = 0
FV = ?
P / Y = 1
C / Y = 1
PMT : END

(A1) for correct values

The real value of amount of interest

$= 21537.04494 - 8500$ (M1) for valid approach

$= 13037.04494$

$= 13000 \text{ EUR}$ A1 N3

[3]

3. (a) Let $r\%$ be the real interest rate per year.

$2800\left(1+\frac{r}{100}\right)^{12}=4000$ (M1)(A1) for substitution

$2800\left(1+\frac{r}{100}\right)^{12}-4000=0$

By considering the graph of

$y=2800\left(1+\frac{r}{100}\right)^{12}-4000$, $r=3.016904692$.

Thus, $r=3.02$. A1 N3

By TVM Solver :
N = 12
I% = ?
PV = –2800
PMT = 0
FV = 4000
P / Y = 1
C / Y = 1
PMT : END

(M1)(A1) for correct values

Thus, $r=3.02$. A1 N3

[3]

(b) The rate of inflation per year

$=4\%-3.016904692\%$ (M1) for valid approach

$=0.9830953083\%$

$=0.983\%$ A1 N2

[2]

4. (a) Let $r\%$ be the nominal annual interest rate compounded yearly.

$$(1+r\%)^8 = \left(1+\frac{9.2}{(100)(4)}\right)^{(4)(8)}$$ (M1)(A1) for substitution

$1+r\% = 1.023^4$

$r = 9.522294784$

The real interest rate per year

$= 9.522294784\% - i\%$ (M1) for valid approach

$= (9.5223 - i)\%$ A1 N4

[4]

(b) $14500\left(1+\frac{9.5223-i}{100}\right)^8 = 18500$ (M1)(A1) for substitution

$14500\left(1+\frac{9.5223-i}{100}\right)^8 - 18500 = 0$

By considering the graph of

$y = 14500\left(1+\frac{9.5223-i}{100}\right)^8 - 18500$,

$i = 6.4301811$.

Thus, $i = 6.43$. A1 N3

By TVM Solver :
N = 8
I% = ?
PV = −14500
PMT = 0
FV = 18500
P / Y = 1
C / Y = 1
PMT : END

(M1)(A1) for correct values

$i = 9.5223 - 3.092118852$

$i = 6.430181148$

$i = 6.43$ A1 N3

[3]

Exercise 26

1. (a) By TVM Solver:

N = 20
I% = 7.5
PV = 0
PMT = ?
FV = 60000
P / Y = 1
C / Y = 1
PMT : BEGIN

(M1)(A1) for correct values

PV = –1288.86651

Thus, the value of the regular payment per year is \$1290. A1 N3

[3]

(b) By TVM Solver:

N = ?
I% = 7.5
PV = 0
PMT = –1788.86651
FV = 60000
P / Y = 1
C / Y = 1
PMT : BEGIN

(M1)(A1) for correct values

N = 16.67555757

Thus, the number of years required is 16.7 years. A1 N3

[3]

2. (a) By TVM Solver:

$N = 5 \times 12$
$I\% = 3$
$PV = 0$
$PMT = -1000$
$FV = ?$
$P/Y = 12$
$C/Y = 1$
$PMT : END$

(M1)(A1) for correct values

$FV = 64580.96194$

Thus, the value of the investment after five years is \$64600. A1 N3

[3]

(b) By TVM Solver:

$N = 5 \times 12$
$I\% = 3$
$PV = 0$
$PMT = -1500$
$FV = ?$
$P/Y = 12$
$C/Y = 1$
$PMT : END$

(M1)(A1) for correct values

$FV = 96871.44291$

The value of the investment after ten years

$= 64580.96194 \times (1+3\%)^5 + 96871.44291$

$= 171738.4778$

$= \$172000$ A1 N3

[3]

3. (a) By TVM Solver:

$N = 15 \times 4$
$I\% = 5$
$PV = 0$
$PMT = -300$
$FV = ?$
$P/Y = 4$
$C/Y = 1$
$PMT : END$

(M1)(A1) for correct values

$FV = 26374.85909$

Thus, the value of the investment after fifteen years is \$26400. A1 N3

[3]

(b) By TVM Solver:

$N = 30 \times 4$
$I\% = 5$
$PV = 0$
$PMT = ?$
$FV = 3.5 \times 26374.85909$
$P/Y = 4$
$C/Y = 1$
$PMT : END$

(M1)(A1) for correct values

$PMT = -341.0277664$

Thus, the new amount of deposit is \$341. A1 N3

[3]

4. (a) By TVM Solver:

$N = 8 \times 12$
$I\% = 2.9$
$PV = 0$
$PMT = -100$
$FV = ?$
$P/Y = 12$
$C/Y = 1$
$PMT : BEGIN$

(M1)(A1) for correct values

$FV = 10799.30951$

Thus, the value of the investment after eight years for annuity X is \$10800. A1 N3

[3]

(b) By TVM Solver:

$N = 8 \times 12$
$I\% = 2.9$
$PV = 0$
$PMT = -200$
$FV = ?$
$P/Y = 12$
$C/Y = 1$
$PMT : BEGIN$

(M1)(A1) for correct values

$FV = 21598.61903$

The value of the investment after sixteen years for annuity X

$= 10799.30951 \times (1 + 2.9\%)^8 + 21598.61903$

$= 35172.96727$

$= \$35200$ A1 N3

[3]

(c) By TVM Solver:

$N = 16 \times 12$
$I\% = 2.9$
$PV = 0$
$PMT = ?$
$FV = 35172.96727$
$P/Y = 12$
$C/Y = 1$
$PMT : BEGIN$

(A1) for correct values

$PMT = -144.3072994$

Thus, $p = 144$. A1 N2

[2]

Exercise 27

1. (a) (i) By TVM Solver:

N = 144
I% = 3.7
PV = 1900000
PMT = ?
FV = 0
P / Y = 12
C / Y = 1
PMT : END

(M1)(A1) for correct values

PMT = −16303.73311

Thus, the amount of monthly payment is \$16300. A1 N3

(ii) The total amount to be paid

= (16303.73311)(144) (M1) for valid approach

= 2347737.568

= \$2350000 A1 N2

(iii) The amount of interest paid

= 2347737.568 − 1900000 (M1) for valid approach

= 447737.5678

= \$448000 A1 N2

[7]

(b) (i) By TVM Solver:

N = ?
I% = 3.4
PV = 1900000 − 350000
PMT = −17500
FV = 0
P / Y = 12
C / Y = 1
PMT : END

(M1)(A1) for correct values

N = 101.8779513

Thus, the number of months to repay the loan is 102 months. A1 N3

(ii) The total amount to be paid

= 350000 + (17500)(102) (M1) for valid approach

= \$2135000 A1 N2

(iii) The amount of interest paid

$= 2135000 - 1900000$ (M1) for valid approach

$= \$235000$ A1 N2

[7]

(c) The amount of monthly payment in option 1 is less than that in option 2. R1

Thus, the option 1 is better. A1 N2

[2]

(d) The amount of interest paid in option 2 is less than that in option 1. R1

Thus, the option 2 is better. A1 N2

[2]

2. (a) (i) By TVM Solver:

$N = 36$
$I\% = 4.5$
$PV = 40000 - 10000$
$PMT = ?$
$FV = 0$
$P/Y = 12$
$C/Y = 1$
$PMT : END$

(M1)(A1) for correct values

$PMT = -891.1985089$

Thus, the amount of monthly payment is \$891. A1 N3

(ii) The amount of interest paid

$= (891.1985089)(36) + 10000 - 40000$ (M1)(A1) for substitution

$= 2083.146321$

$= \$2080$ A1 N3

[6]

(b) (i) By TVM Solver:

$N = ?$
$I\% = 4.5$
$PV = 40000$
$PMT = -800$
$FV = 0$
$P/Y = 12$
$C/Y = 1$
$PMT : END$

(M1)(A1) for correct values

$N = 55.34864756$

Thus, the number of months to repay the loan is 56 months. A1 N3

(ii) The amount of interest paid

$= (800)(56) - 40000$ (M1)(A1) for substitution

$= \$4800$ A1 N3

[6]

(c) The amount of monthly payment in option 2 is less than that in option 1. R1

Thus, the option 2 is better. A1 N2

[2]

(d) The amount of interest paid in option 1 is less than that in option 2. R1

Thus, the option 1 is better. A1 N2

[2]

(e) By TVM Solver:

N = 60
I% = ?
PV = 40000
PMT = –900
FV = 0
P / Y = 12
C / Y = 1
PMT : END

(M1)(A1) for correct values

I% = 13.24614765

Thus, $r = 13.2$. A1 N3

[3]

3. (a) (i) By TVM Solver:

N = 120
I% = 2
PV = 10000
PMT = ?
FV = 0
P / Y = 12
C / Y = 1
PMT : END

(M1)(A1) for correct values

PMT = –91.93240592

Thus, the amount of monthly payment is \$91.9. A1 N3

(ii) The amount of interest paid

$= (91.93240592)(120) - 10000$ (M1)(A1) for substitution

$= 1031.88871$

$= \$1030$ A1 N3

[6]

(b) (i) By TVM Solver:

N = 60
I% = 2
PV = 10000
PMT = –91.93240592
FV = ?
P / Y = 12
C / Y = 1
PMT : END

(M1)(A1) for correct values

FV = –5247.330813

Thus, the amount of the loan after 5 years is \$5247.330813.

By TVM Solver:

N = ?
I% = 2
PV = 5247.330813
PMT = –91.93240592 – 60
FV = 0
P / Y = 12
C / Y = 1
PMT : END

(M1)(A1) for correct values

N = 35.59078942

Thus, the number of months to repay the loan is 96 months. A1 N5

(ii) The amount of interest paid

$= (91.93240592)(60)$
$+(91.93240592+60)(36)-10000$
$= 985.5109683$
$= \$986$

(M1)(A1) for substitution

A1 N3

(iii) The amount of interest paid in option 2 is less than that in option 1. R1 N1

[9]

(c) (i) By TVM Solver:

N = ?
I% = 2
PV = 10000
PMT = –91.93240592×1.5
FV = 0
P / Y = 12
C / Y = 1
PMT : END

(M1)(A1) for correct values

N = 77.30461672

Thus, the number of months to repay the loan is 78 months. A1 N3

(ii) 18 months A1 N1

[4]

4. (a) (i) By TVM Solver:

N = 20
I% = 2
PV = 50000
PMT = ?
FV = 0
P / Y = 1
C / Y = 1
PMT : BEGIN

(M1)(A1) for correct values

PMT = –2997.878339

Thus, $R_1 = 3000$. A1 N3

(ii) By TVM Solver:

N = 20
I% = 2
PV = 50000
PMT = ?
FV = 0
P / Y = 1
C / Y = 1
PMT : END

(M1)(A1) for correct values

PMT = –3057.835906

Thus, $R_2 = 3060$. A1 N3

(iii) The difference between the total amounts to be paid for the version 1 and the version 2 A1 N1

(iv) Version 1 A1 N1

[8]

(b) (i) By TVM Solver:

N = 240
I% = 2
PV = 50000
PMT = ?
FV = 0
P / Y = 12
C / Y = 1
PMT : END

(M1)(A1) for correct values

PMT = –252.5132304

Thus, $R_3 = 253$. A1 N3

(ii) The amount of interest paid in version 3 A1 N1

(iii) The amount of interest paid in version 2

$= (3057.835906)(20) - 50000$ (M1) for valid approach

$= \$11156.71812$

The amount of interest paid in version 3

$= (252.5132304)(240) - 50000$ (M1) for valid approach

$= \$10603.1753$

Hence, the version 3 will have the smaller total amount to be paid. A1 N3

[7]

Chapter 10 Solution

Exercise 28

1. (a) The coordinates of M

$= \left(\frac{-14+0}{2}, \frac{-48+0}{2}, \frac{0+0}{2} \right)$ (A1) for substitution

$= (-7, -24, 0)$ A1 N2

[2]

(b) (i) $(-7, -24, n)$ A1 N1

(ii) $\sqrt{(-7-0)^2 + (-24-0)^2 + (n-0)^2} = 65$ (M1) for setting equation

$\sqrt{625+n^2} = 65$

$\sqrt{625+n^2} - 65 = 0$

By considering the graph of

$\sqrt{625+n^2} - 65 = 0$, $n = -60$ (*Rejected*) or

$n = 60$. A1 N2

[3]

2. (a) The length of PQ

$= \sqrt{(45-15)^2 + (85-25)^2 + (15-35)^2}$ (A1) for substitution

$= 70$ A1 N2

[2]

(b) The coordinates of M

$= \left(\frac{15+45}{2}, \frac{25+85}{2}, \frac{35+15}{2} \right)$ (A1) for substitution

$= (30, 55, 25)$ A1 N2

[2]

(c) (i) $(30, 55, 5)$ A1 N1

(ii) The length of QN

$= \sqrt{(45-30)^2 + (85-55)^2 + (15-5)^2}$ (A1) for substitution

$= 35$ A1 N2

[3]

3. (a) The coordinates of M

$= \left(\frac{12+(-24)}{2}, \frac{-2+30}{2}, \frac{-8+4}{2} \right)$ (A1) for substitution

$= (-6, 14, -2)$ A1 N2

[2]

(b) The length of MN

$= \sqrt{40^2 + 9^2}$ (A1) for substitution

$= 41$ A1 N2

[2]

(c) $\tan \hat{PMN} = \frac{30.75}{41}$ (M1) for tangent ratio

$\hat{PMN} = 36.86989765^\circ$

Thus, the required angle of elevation is 36.9°. A1 N2

[2]

4. (a) $\sqrt{(29-(-16))^2 + (-100-8)^2 + (h-24)^2} = 195$ (M1) for setting equation

$\sqrt{13689 + (h-24)^2} = 195$

$\sqrt{13689 + (h-24)^2} - 195 = 0$

By considering the graph of

$y = \sqrt{13689 + (h-24)^2} - 195$, $h = -132$ or

$h = 180$ (*Rejected*). A1 N2

[2]

(b) $\sin \hat{QPR} = \frac{156}{195}$ (M1) for sine ratio

$\hat{QPR} = 53.13010235^\circ$

Thus, the required angle of depression is 53.1°. A1 N2

[2]

(c) $\left(\frac{29+x}{2}, \frac{-100+y}{2}, \frac{-132+z}{2} \right) = (-16, 8, 24)$ (M1) for valid approach

$x = -61$, $y = 116$ and $z = 180$

Thus, the coordinates of S are $(-61, 116, 180)$. A1 N2

[2]

Exercise 29

1. (a) The gradient of L

$= \dfrac{11-6}{20-10}$ (M1) for valid approach

$= \dfrac{1}{2}$

The equation of L:

$y-11=\dfrac{1}{2}(x-20)$ A1

$2y-22=x-20$

$x-2y+2=0$ A1 N3

[3]

(b) The x-intercept of L is -2 A1

The y-intercept of L is 1 A1 N2

[2]

(c) $(-1, 0.5)$ A1 N1

[1]

2. (a) The gradient of L

$= \dfrac{-26-(-8)}{2-(-4)}$ (M1) for valid approach

$=-3$

The equation of L:

$y+8=-3(x+4)$ A1

$y+8=-3x-12$

$3x+y+20=0$ A1 N3

[3]

(b) The x-intercept of L is $-\dfrac{20}{3}$ A1

The y-intercept of L is -20 A1 N2

[2]

(c) $\dfrac{1}{3}$ A1 N1

[1]

3. (a) The gradient of L_1

$= \frac{37-1}{17-5}$ (M1) for valid approach

$= 3$

The equation of L_1:

$y-1=3(x-5)$ A1

$y-1=3x-15$

$3x-y-14=0$ A1 N3

[3]

(b) The gradient of L_2

$= -\frac{3}{-1}$ A1

$= 3$

As the gradients of L_1 and L_2 are the same,

L_1 and L_2 are parallel. R1 N2

[2]

4. (a) The gradient of L_1

$= \frac{40-0}{4-(-4)}$ (M1) for valid approach

$= 5$

The equation of L_1:

$y-0=5(x+4)$ A1

$y=5x+20$

$5x-y+20=0$ A1 N3

[3]

(b) The gradient of L_2

$= -\frac{1}{5}$ A1

The product of slopes

$= 5\times -\frac{1}{5}$

$= -1$

Thus, L_1 and L_2 are perpendicular. R1 N2

[2]

Exercise 30

1. (a) The gradient of L_1 is $\frac{1}{2}$ A1

The y-intercept of L_1 is 8 A1 N2

[2]

(b) The gradient of L_2 is $\frac{1}{2}$ (A1) for correct value

The equation of L_2:

$y-5=\frac{1}{2}(x+2)$ A1

$2y-10=x+2$

$x-2y+12=0$ A1 N3

[3]

2. (a) The gradient of L_1 is $-\frac{3}{2}$ A1 N1

[1]

(b) $3(4)+2a-4=0$ (M1) for substitution

$2a+8=0$

$2a=-8$

$a=-4$ A1 N2

[2]

(c) The gradient of L_2 is $-\frac{3}{2}$ (A1) for correct value

The equation of L_2:

$y+7=-\frac{3}{2}(x-1)$ A1

$2y+14=3-3x$

$3x+2y+11=0$ A1 N3

[3]

3. (a) The gradient of L_1 is -3 — A1

The x-intercept of L_1 is -7 — A1 N2

[2]

(b) The gradient of L_2 is $\frac{1}{3}$ — (A1) for correct value

The equation of L_2:

$y-0=\frac{1}{3}(x+7)$ — A1

$3y=x+7$

$x-3y+7=0$ — A1 N3

[3]

4. (a) (i) $\frac{1}{2}$ — A1 N1

(ii) $-\frac{17}{4}$ — A1 N1

[2]

(b) The gradient of L_2 is -2 — (A1) for correct value

The equation of L_2:

$y+\frac{17}{4}=-2(x-0)$ — A1

$4y+17=-8x$

$8x+4y+17=0$ — A1 N3

[3]

(c) $8b+4(5.75)+17=0$ — (M1) for substitution

$8b+40=0$

$8b=-40$

$b=-5$ — A1 N2

[2]

Exercise 31

1. (a) The gradient of L_1

$= \dfrac{6-0}{-4-(-2)}$ (M1) for valid approach

$= -3$ A1 N2

[2]

(b) The equation of L_1:

$y-0=-3(x-(-2))$ (M1) for substitution

$y=-3x-6$ (A1) for simplification

$3x+y+6=0$ A1 N3

[3]

(c) The coordinates of C are $(2, 0)$. (A1) for correct values

The equation of L_2:

$y-0=-3(x-2)$ (M1) for substitution

$y=-3x+6$ A1 N3

[3]

(d) The coordinates of D

$= \left(\dfrac{-4+2}{2}, \dfrac{6+0}{2}\right)$ (A1) for substitution

$= (-1, 3)$ A1 N2

[2]

(e) The gradient of L_3

$= -1 \div -3$

$= \dfrac{1}{3}$ (A1) for correct value

The equation of L_3:

$y-3=\dfrac{1}{3}(x-(-1))$ (M1) for substitution

$3y-9=x+1$

$x-3y+10=0$ A1 N3

[3]

(f) $\text{CD} = \frac{k}{\sqrt{5}}\text{BD}$

$\sqrt{(-2-(-1))^2+(0-3)^2}$

$= \frac{k}{\sqrt{5}}\left(\sqrt{(2-(-1))^2+(0-3)^2}\right)$ (M1)(A1) for substitution

$\sqrt{18} = \frac{k}{\sqrt{5}}(\sqrt{10})$ (A1) for simplification

$k = 3$ A1 N4

[4]

2. (a) $\frac{k+30}{2}=25$ (M1) for setting equation

$k+30=50$

$k=20$ A1 N2

[2]

(b) The gradient of L_1

$=\frac{30-25}{40-20}$ (M1) for valid approach

$=\frac{1}{4}$ A1 N2

[2]

(c) The equation of L_1:

$y-20=\frac{1}{4}(x-0)$ (M1) for substitution

$y=\frac{1}{4}x+20$ A1 N2

[2]

(d) The gradient of L_2

$=-1\div\frac{1}{4}$

$=-4$ (A1) for correct value

The equation of L_2:

$y-20=-4(x-0)$ (M1) for substitution

$y=-4x+20$ A1 N3

[3]

(e) The equation of L_3:

$y-0=-4(x-20)$ (M1) for substitution

$y=-4x+80$ (A1) for simplification

$4x+y-80=0$ A1 N3

[3]

(f) $4r+r-80=0$ (M1) for substitution

$5r=80$

$r=16$ A1 N2

[2]

(g) $4x+\left(\frac{1}{4}x+20\right)-80=0$ (M1) for substitution

$\frac{17}{4}x=60$

$x=\frac{240}{17}$

$y=\frac{1}{4}\left(\frac{240}{17}\right)+20$ (M1) for substitution

$y=\frac{400}{17}$

Thus, the coordinates of D are $\left(\frac{240}{17},\frac{400}{17}\right)$. A1 N3

[3]

3. (a) The gradient of L_1

$= \dfrac{0-2k}{3k-0}$ (M1) for valid approach

$= \dfrac{-2k}{3k}$

$= -\dfrac{2}{3}$ A1 N2

[2]

(b) The equation of L_1:

$y-2k = -\dfrac{2}{3}(x-0)$ (M1) for substitution

$3y-6k = -2x$ (A1) for simplification

$2x+3y-6k = 0$ A1 N3

[3]

(c) $2(-30)+3(40)-6k = 0$ (M1) for substitution

$60 = 6k$

$k = 10$ A1 N2

[2]

(d) The gradient of L_2

$= -1 \div -\dfrac{2}{3}$

$= \dfrac{3}{2}$ (A1) for correct value

The equation of L_2:

$y-(-2.5) = \dfrac{3}{2}(x-15)$ (M1) for substitution

$2y+5 = 3x-45$

$3x-2y-50 = 0$ A1 N3

[3]

(e) $h = 15,\ k = -2.5$ A2 N2

[2]

(f) $20 = a(0-15)^2 - 2.5$ (M1) for setting equation

$22.5 = 225a$

$a = 0.1$ A1 N2

[2]

(g) $0.1(x-15)^2-2.5=0$ (M1) for setting equation

$(x-15)^2=25$

$x-15=\sqrt{25}$ or $x-15=-\sqrt{25}$

$x=20$ or $x=10$

Thus, the x-intercepts are 10 and 20. A2 N3

[3]

4. (a) $2x^2+4x-16=0$ (M1) for setting equation

$x^2+2x-8=0$

$(x+4)(x-2)=0$

$x=-4$ or $x=2$

$\therefore a=-4,\ b=2$ A2 N3

[3]

(b) -16 A1 N1

[1]

(c) $h=-1,\ k=-18$ A2 N2

[2]

(d) The gradient of VB

$=\dfrac{-18-0}{-1-2}$ (M1) for valid approach

$=6$ A1 N2

[2]

(e) The equation of VB:

$y-0=6(x-2)$ (M1) for substitution

$y=6x-12$ (A1) for simplification

$6x-y-12=0$ A1 N3

[3]

(f) (i) The gradient of CD

$=-1\div 6$

$=-\dfrac{1}{6}$ (A1) for correct value

$\dfrac{0-(-16)}{d-0}=-\dfrac{1}{6}$ (M1) for setting equation

$\dfrac{16}{d}=-\dfrac{1}{6}$

$d=-96$ A1 N3

(ii) $\dfrac{0-(-16)}{d-0}=6$ (M1) for setting equation

$\dfrac{16}{d}=6$

$d=\dfrac{8}{3}$ A1 N2

[5]

Exercise 32

1. (a) The gradient of L_1

$= \dfrac{0-100}{200-0}$ (M1) for valid approach

$= -\dfrac{1}{2}$ A1 N2

[2]

(b) The equation of L_1:

$y-100 = -\dfrac{1}{2}(x-0)$ (M1) for substitution

$2y-200 = -x$ (A1) for simplification

$x+2y-200 = 0$ A1 N3

[3]

(c) $m = 2$, $c = 0$ A2 N2

[2]

(d) $x+2(2x)-200 = 0$ (M1) for substitution

$5x = 200$

$x = 40$

$y = 2(40)$ (M1) for substitution

$y = 80$

Thus, the coordinates of C are $(40, 80)$. A1 N3

[3]

(e) The area of the triangle OAC

$= \dfrac{(100-0)(40-0)}{2}$ (A1) for correct formula

$= 2000$ A1 N2

[2]

(f) The area of the triangle OBC

$= \dfrac{(200-0)(80-0)}{2}$

$= 8000$ (A1) for correct value

$\therefore \dfrac{8000}{2000} = \dfrac{r}{1}$ (M1) for valid approach

$r = 4$ A1 N3

[3]

2. (a) The gradient of L_1

$= \dfrac{0-20}{-60-(-20)}$ (M1) for valid approach

$= \dfrac{1}{2}$ A1 N2

[2]

(b) The equation of L_1:

$y-0=\dfrac{1}{2}(x-(-60))$ (M1) for substitution

$2y = x+60$ (A1) for simplification

$x-2y+60=0$ A1 N3

[3]

(c) The equation of L_2:

$y-0=\dfrac{1}{2}(x-(-30))$ (M1) for substitution

$y=\dfrac{1}{2}x+15$ A1 N2

[2]

(d) The gradient of L_3

$= -1 \div \dfrac{1}{2}$

$= -2$ (A1) for correct value

The equation of L_3:

$y-20=-2(x-(-20))$ (M1) for substitution

$y=-2x-20$ A1 N3

[3]

(e) $\dfrac{1}{2}x+15=-2x-20$ (M1) for substitution

$\dfrac{5}{2}x=-35$

$x=-14$

$y=-2(-14)-20$ (M1) for substitution

$y=8$

Thus, the coordinates of D are $(-14, 8)$. A1 N3

[3]

(f) $0 = -2x - 20$ (M1) for substitution

$2x = -20$

$x = -10$

Thus, the coordinates of E are $(-10, 0)$. (A1) for correct values

The area of the triangle CDE

$= \frac{(-10-(-30))(8-0)}{2}$ (A1) for correct formula

$= 80$ A1 N4

[4]

3. (a) $\frac{a-0}{0-30}=-\frac{4}{3}$ (M1) for setting equation

$-\frac{a}{30}=-\frac{4}{3}$

$a=40$ A1 N2

[2]

(b) The equation of L_1:

$y-0=-\frac{4}{3}(x-30)$ (M1) for substitution

$3y=-4x+120$ (A1) for simplification

$4x+3y-120=0$ A1 N3

[3]

(c) The gradient of L_2

$=-1\div-\frac{4}{3}$

$=\frac{3}{4}$ (A1) for correct value

The equation of L_2:

$y-0=\frac{3}{4}(x-30)$ (M1) for substitution

$y=\frac{3}{4}x-\frac{45}{2}$ A1 N3

[3]

(d) $c=\frac{3}{4}(3c)-\frac{45}{2}$ (M1) for substitution

$-\frac{5}{4}c=-\frac{45}{2}$

$c=18$ A1 N2

[2]

(e) $CE^2=DE^2-CD^2$ (M1) for setting equation

$CE^2=(15\sqrt{13})^2-30^2$ (A1) for substitution

$CE^2=2025$

$CE=45$ A1 N3

[3]

(f) The area of the triangle CDE

$=\frac{(45)(30)}{2}$ (M1) for valid approach

$=675$ A1 N2

[2]

4. (a) The gradient of L_1

$= \dfrac{10-0}{-20-0}$ (M1) for valid approach

$= -\dfrac{1}{2}$ A1 N2

[2]

(b) $p = 20$, $q = -10$ A2 N2

[2]

(c) The equation of L_1:

$y - 0 = -\dfrac{1}{2}(x-0)$ (M1) for substitution

$y = -\dfrac{1}{2}x$ A1 N2

[2]

(d) The gradient of L_2

$= -1 \div -\dfrac{1}{2}$

$= 2$ (A1) for correct value

The equation of L_2:

$y - 0 = 2(x-0)$ (M1) for substitution

$2x - y = 0$ A1 N3

[3]

(e) $\text{AB} = \sqrt{(-20-20)^2 + (10-(-10))^2}$ (M1) for valid approach

$\text{AB} = \sqrt{2000}$

$\dfrac{(\text{AB})(\text{OC})}{2} = 250$ (M1) for setting equation

$\dfrac{(\sqrt{2000})(\text{OC})}{2} = 250$

$\text{OC} = \sqrt{125}$ A1 N3

[3]

(f) Let $(c, 2c)$ be the coordinates of C.

$\sqrt{(c-0)^2 + (2c-0)^2} = \sqrt{125}$ (M1) for setting equation

$\sqrt{5c^2} = \sqrt{125}$

$5c^2 = 125$ (A1) for correct approach

$c^2 = 25$

$c = -5$ (*Rejected*) or $c = 5$ (A1) for correct value

Thus, the coordinates of C are $(5, 10)$. A1 N4

[4]

Chapter 11 Solution

Exercise 33

1. (a) The coordinates of C

$= \left(\frac{0+0}{2}, \frac{6+24}{2} \right)$ (A1) for substitution

$= (0, 15)$ A1 N2

[2]

(b) 0 A1 N1

[1]

(c) $y = 15$ A1 N1

[1]

(d) The distance between A and D is equal to the distance between B and D. A1 N1

[1]

2. (a) (i) (100, 0) A1 N1

(ii) (0, 50) A1 N1

(iii) The coordinates of C

$= \left(\frac{100+0}{2}, \frac{0+50}{2} \right)$ (A1) for substitution

$= (50, 25)$ A1 N2

[4]

(b) 2 A1 N1

[1]

(c) The equation of the perpendicular bisector of AB:

$y - 25 = 2(x - 50)$ (M1) for substitution

$y - 25 = 2x - 100$

$y = 2x - 75$ A1 N2

[2]

3. (a) (i) $\frac{3}{4}$ A1 N1

(ii) The equation of AB:

$y-(-6)=\frac{3}{4}(x-12)$ (M1) for substitution

$y=\frac{3}{4}x-15$ A1 N2

[3]

(b) $\frac{3}{4}x-15=-\frac{4}{3}x+35$ (M1) for substitution

$9x-180=-16x+420$

$25x=600$

$x=24$

$y=\frac{3}{4}(24)-15$ (M1) for substitution

$y=3$

Thus, the required coordinates are $(24, 3)$. A1 N3

[3]

(c) Let (a, b) be the coordinates of A.

$\frac{a+12}{2}=24$

$a=36$

$\frac{b+(-6)}{2}=3$

$b=12$

Thus, the coordinates of A are $(36, 12)$. A1 N2

[2]

4. (a) (i) $\frac{4}{3}$ A1 N1

(ii) The equation of AB:

$y-20=\frac{4}{3}(x-0)$ (M1) for substitution

$y=\frac{4}{3}x+20$ A1 N2

[3]

(b) $\frac{4}{3}b+20$ A1 N1

[1]

(c) (i) 25 A1 N1

(ii) BC = AC

$\sqrt{(b-25)^2+\left(\left(\frac{4}{3}b+20\right)-20\right)^2}=25$ (A1) for correct equation

$\sqrt{(b-25)^2+\frac{16}{9}b^2}-25=0$

By considering the graph of

$y=\sqrt{(b-25)^2+\frac{16}{9}b^2}-25,\ b=0$ (*Rejected*)

or $b=18$. A1 N2

[3]

Exercise 34

1. (a) The required area

$= \frac{(3+8)(5)}{2}$ (M1) for valid approach

$= 27.5\ \text{km}^2$ A1 N2

[2]

(b) (i) The gradient of L

$= \frac{7-5}{5-1}$ (M1) for valid approach

$= \frac{1}{2}$ A1 N2

(ii) The equation of L:

$y-5=\frac{1}{2}(x-1)$ (M1) for substitution

$y=\frac{1}{2}x+\frac{9}{2}$ A1 N2

[4]

(c) So Yeon's home is on the boundary separating the Voronoi cells of the restaurant B and the restaurant C, which is equidistant to them. A1 N1

[1]

2. (a) (i) The gradient of L

$= \frac{10-7}{6-5}$ (M1) for valid approach

$= 3$ A1 N2

(ii) The equation of L:

$y-7=3(x-5)$ (M1) for substitution

$y=3x-8$ A1 N2

[4]

(b) The police station B A1 N1

[1]

(c) $5 < x \leq 10$ A1 N1

[1]

3. (a) (i) The required distance

$= \sqrt{(4.15-4)^2 + (4-2)^2}$ (M1) for valid approach

$= 2.005617112$

$= 2.01$ A1 N2

(ii) 2.15 A1 N1

(iii) 1.85 A1 N1

[4]

(b) The school C A1 N1

[1]

4. (a) The mid-point of AC

$= \left(\frac{2.5+7.5}{2}, \frac{2.5+7.5}{2} \right)$ (A1) for substitution

$= (5, 5)$ A1 N2

[2]

(b) $2.5 < x < 5$ A1 N1

[1]

(c) The distance from the park to the fire station A

$= \sqrt{(5.5-2.5)^2 + (6.5-2.5)^2}$ M1

$= 5$

The distance from the park to the fire station B

$= \sqrt{(5.5-7.5)^2 + (6.5-7.5)^2}$ M1

$= 2.236067977$

< 5 R1

Thus, the park lies in the Voronoi cell of the fire station B. AG N0

[3]

Exercise 35

1. (a) $x = 5$ A1 N1

[1]

(b) (i) $(7, 4)$ A1 N1

(ii) $7 - 2(4) + k = 0$ (M1) for substitution

$k = 1$ A1 N2

(iii) $5 - 2y + 1 = 0$ (M1) for substitution

$6 = 2y$

$y = 3$

Thus, the required coordinates are $(5, 3)$. A1 N2

[5]

2. (a) (i) The mid-point of AB

$= \left(\frac{4+8}{2}, \frac{8+4}{2} \right)$ (A1) for substitution

$= (6, 6)$ A1 N2

(ii) $y = x$ A1 N1

[3]

(b) (i) $x = 5$ A1 N1

(ii) $(5, 5)$ A1 N1

(iii) The radius

$= \sqrt{(5-2)^2 + (5-4)^2}$ (M1) for valid approach

$= 3.16227766$

$= 3.16$ A1 N2

[4]

3. (a) (i) The mid-point of AB

$= \left(\frac{2+8}{2}, \frac{2+8}{2} \right)$ (A1) for substitution

$= (5, 5)$ A1 N2

(ii) $5 + 5 = k$ (M1) for substitution

$k = 10$ A1 N2

[4]

(b) $x + (-0.5x + 9.5) = 10$ (M1) for substitution

$0.5x = 0.5$

$x = 1$

$1 + y = 10$ (M1) for substitution

$y = 9$

Thus, the required coordinates are $(1, 9)$. A1 N3

[3]

(c) Every position in the Voronoi cell of B has B to be the nearest supermarket. A1 N1

[1]

4. (a) (i) $x = 5$ A1 N1

(ii) $y = 5$ A1 N1

[2]

(b) (i) (6, 6) A1 N1

(ii) $\frac{k-6}{5-6} = \frac{k-5}{5-k}$ (A1) for correct equation

$\frac{k-6}{-1} = -1$

$k - 6 = 1$

$k = 7$ A1 N2

(iii) The decrease in the area

$= \frac{(7-5)(7-5)}{2}$ (M1)(A1) for correct formula

$= 2$ A1 N3

[6]

Exercise 36

1. (a) (i) The radius

$= \sqrt{(5-4.125)^2 + (8-5)^2}$ (M1) for valid approach

$= 3.125$ km A1 N2

(ii) 3 km A1 N1

(iii) E A1 N1

[4]

(b) 13 km A1 N1

[1]

2. (a) (i) The radius

$= \sqrt{(7-5)^2 + (8-7.75)^2}$ (M1) for valid approach

$= 2.015564437$

$= 2.02$ km A1 N2

(ii) The radius

$= \sqrt{(5-4)^2 + (4.5-3)^2}$ (M1) for valid approach

$= 1.802775638$

$= 1.80$ km A1 N2

(iii) The hotel at E A1 N1

[5]

(b) The difference

$= 2\pi(2.015564437) - 2\pi(1.802775638)$ (M1) for valid approach

$= 1.336991457$

$= 1.34$ km A1 N2

[2]

3. (a) (i) The radius

$= \sqrt{(4-2)^2 + (5-2)^2}$ (M1) for valid approach

$= 3.605551275$

$= 3.61$ km A1 N2

(ii) The radius

$= \sqrt{(8-5)^2 + \left(8 - \frac{17}{3}\right)^2}$ (M1) for valid approach

$= 3.80058475$

$= 3.80$ km A1 N2

(iii) Q A1 N1

[5]

(b) The farm at B A1 N1

[1]

4. (a) The radius

$= \sqrt{\left(\frac{14}{3} - 2\right)^2 + \left(\frac{9}{2} - 1\right)^2}$ (M1) for valid approach

$= 4.400126261$

$= 4.40$ km A1 N2

[2]

(b) 4 km A1 N1

[1]

(c) The percentage error

$= \left| \frac{4 - \frac{\sqrt{185}}{4}}{\frac{\sqrt{185}}{4}} \right| \times 100\%$ (A1) for correct substitution

$= 17.63433954\%$

$= 17.6\%$ A1 N2

[2]

Chapter 12 Solution

Exercise 37

1. (a) $p = \dfrac{2-(-6)}{2}$ (M1) for valid approach

$p = 4$ A1 N2

[2]

(b) The period of the graph is 180°.

$q = \dfrac{360^\circ}{180^\circ}$ (M1) for valid approach

$q = 2$ A1 N2

[2]

(c) $r = \dfrac{2+(-6)}{2}$ (M1) for valid approach

$r = -2$ A1 N2

[2]

2. (a) $p = \dfrac{60-28}{2}$ (M1) for valid approach

$p = 16$ A1 N2

[2]

(b) The period of the graph is 1440°.

$q = \dfrac{360^\circ}{1440^\circ}$ (M1) for valid approach

$q = \dfrac{1}{4}$ A1 N2

[2]

(c) $r = \dfrac{28+60}{2}$ (M1) for valid approach

$r = 44$ A1 N2

[2]

3. (a) $p = \dfrac{2\pi - (-2\pi)}{2}$ (M1) for valid approach

$p = 2\pi$ A1 N2

[2]

(b) The period of the graph is 60°.

$q = \dfrac{360^\circ}{60^\circ}$ (M1) for valid approach

$q = 6$ A1 N2

[2]

(c) $2\pi = 2\pi\cos(6(15^\circ - r))$ (M1) for setting equation

$1 = \cos(90^\circ - 6r)$

$90^\circ - 6r = 0^\circ$ or $90^\circ - 6r = 360^\circ$

$r = 15^\circ$ or $r = -45^\circ$ (*Rejected*) A1 N2

[2]

4. (a) $p = \dfrac{20 - 0}{2}$ (M1) for valid approach

$p = 10$ A1 N2

[2]

(b) The period of the graph is 1080°.

$q = \dfrac{360^\circ}{1080^\circ}$ (M1) for valid approach

$q = \dfrac{1}{3}$ A1 N2

[2]

(c) $20 = 10\cos\left(\dfrac{1}{3}(1080^\circ + r)\right) + 10$ (M1) for setting equation

$10 = 10\cos\left(360^\circ + \dfrac{1}{3}r\right)$

$\cos\left(360^\circ + \dfrac{1}{3}r\right) = 1$

$360^\circ + \dfrac{1}{3}r = 0^\circ$ or $360^\circ + \dfrac{1}{3}r = 360^\circ$

$r = -1080^\circ$ or $r = 0^\circ$ (*Rejected*) A1 N2

[2]

Exercise 38

1. (a) (i) The amplitude of f is 3.5. A1 N1

(ii) The period of f

$= 360^\circ \div 180^\circ$ (M1) for valid approach

$= 2$ A1 N2

[3]

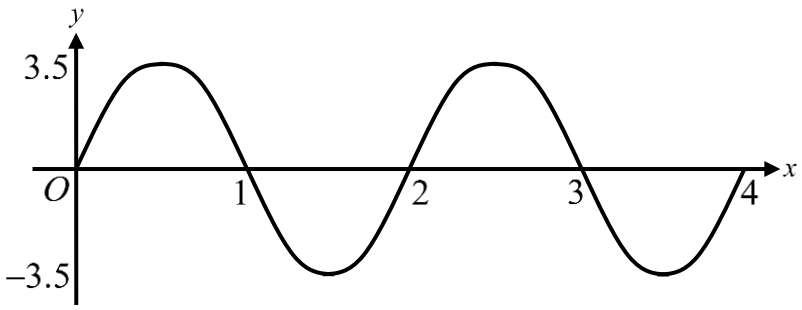
(b) For correct x-intercepts A1

For correct maximum and minimum points A1

For correct domain A1

For sinusoidal curve starting at the origin with correct period A1 N4

[4]



2. (a) (i) The amplitude of f is 3. A1 N1

(ii) The period of f

$= 360^\circ \div 180^\circ$ (M1) for valid approach

$= 2$ A1 N2

[3]

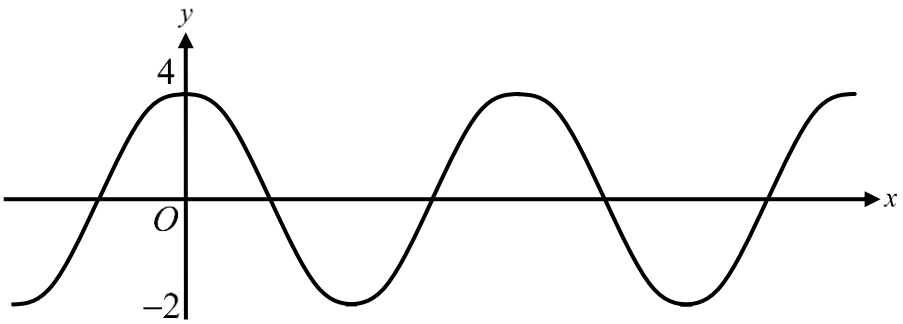
(b) For correct x-intercepts A1

For correct maximum and minimum points A1

For correct domain A1

For sinusoidal curve starting at $(0, 4)$ with correct period A1 N4

[4]



3. (a) (i) The amplitude of f is 4. A1 N1

(ii) The period of f

$= 360^\circ \div 1$ (M1) for valid approach

$= 360^\circ$ A1 N2

[3]

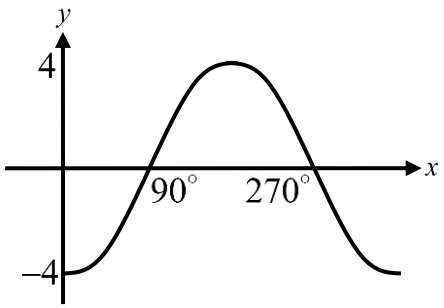
(b) For correct x-intercepts A1

For correct maximum and minimum points A1

For correct domain A1

For sinusoidal curve starting at the $(0, -4)$ with correct period A1 N4

[4]



4. (a) (i) The amplitude of f is 3. A1 N1

(ii) The period of f

$= 360^\circ \div \frac{1}{2}$ (M1) for valid approach

$= 720^\circ$ A1 N2

[3]

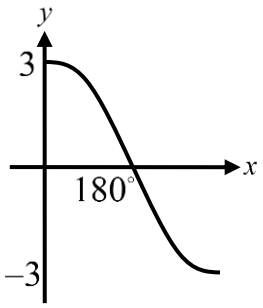
(b) For correct x-intercept A1

For correct maximum and minimum points A1

For correct domain A1

For sinusoidal curve starting at $(0, 3)$ with correct period A1 N4

[4]



Exercise 39

1. (a) The period

$= \frac{360^\circ}{2}$ (M1) for valid approach

$= 180^\circ$ A1 N2

[2]

(b) The minimum value

$= -1 - 1$ (M1) for valid approach

$= -2$ A1 N2

[2]

(c) $f(x) = 0$

$\sin 2x - 1 = 0$

$x = 45^\circ$ or $x = 225^\circ$ A2 N2

[2]

2. (a) 3 A1 N1

[1]

(b) The maximum value

$= 1 - 3(-1)$ (M1) for valid approach

$= 4$ A1 N2

[2]

(c) $f(x) = -2$

$1 - 3\cos x = -2$ (M1) for setting equation

$3 - 3\cos x = 0$

$x = 0^\circ$, $x = 360^\circ$ or $x = 720^\circ$ A2 N3

[3]

3. (a) The period

$= \frac{360^\circ}{4}$ (M1) for valid approach

$= 90^\circ$ A1 N2

[2]

(b) $\{y : 1 \le y \le 5\}$ A2 N2

[2]

(c) $f(x) - 4 = 0$

$2\sin 4x + 3 - 4 = 0$ (M1) for setting equation

$2\sin 4x - 1 = 0$

$x = -82.5^\circ$, $x = -52.5^\circ$, $x = 7.5^\circ$ or $x = 37.5^\circ$ A2 N3

[3]

4. (a) $\frac{1}{2}$ A1 N1

[1]

(b) $\left\{y : -\frac{3}{4} \le y \le \frac{1}{4}\right\}$ A2 N2

[2]

(c) $2f(x)+1=0$

$2\left(-\frac{1}{2}\cos\frac{1}{3}x-\frac{1}{4}\right)+1=0$ (M1) for setting equation

$-\cos\frac{1}{3}x+\frac{1}{2}=0$

$x=180^\circ$ or $x=900^\circ$ A2 N3

[3]

Exercise 40

1. (a) (i) The time required

$= 13.75 - 8.25$ (M1) for valid approach

$= 5.5$ hours A1 N2

(ii) The difference in height

$= 1.8 - 0.4$ (M1) for valid approach

$= 1.4$ m A1 N2

[4]

(b) (i) $p = \dfrac{1.8 - 0.4}{2}$ (M1) for valid approach

$p = 0.7$ A1 N2

(ii) The period

$= 2(5.5)$ (M1) for valid approach

$= 11$ hours (A1) for correct value

$\therefore q = \dfrac{360^\circ}{11}$

$\therefore q = \left(\dfrac{360}{11}\right)^\circ$ A1 N3

(iii) $r = \dfrac{1.8 + 0.4}{2}$ (M1) for valid approach

$r = 1.1$ A1 N2

[7]

(c) Recognizing that 9 April 2018 implies $25 \le t < 49$ (M1) for valid approach

$t = 8.25 + 3(11)$

$t = 41.25$ (A1) for correct value

Thus, the time is 16:15. A1 N3

[3]

2. (a) (i) The time required

$= 2(13 - 6.5)$ (M1) for valid approach

$= 13$ hours A1 N2

(ii) The difference in height

$= 4.2 - 1.8$ (M1) for valid approach

$= 2.4$ m A1 N2

[4]

(b) (i) $p = \dfrac{4.2 - 1.8}{2}$ (M1) for valid approach

$p = 1.2$ A1 N2

(ii) The period is 13 hours. (A1) for correct value

$\therefore q = \dfrac{360^\circ}{13}$

$\therefore q = \left(\dfrac{360}{13}\right)^\circ$ A1 N2

(iii) $r = \dfrac{4.2 + 1.8}{2}$ (M1) for valid approach

$r = 3$ A1 N2

[6]

(c) Recognizing that 25 August 2018 implies

$36 \le t < 60$ (M1) for valid approach

$t = 13 + 3(13)$

$t = 52$ (A1) for correct value

Thus, the time is 16:00. A1 N3

[3]

3. (a) The time required

$= 2 \times 9$ (M1) for valid approach

$= 18$ minutes

Hence, the Ferris wheel will first reach a height of 91 m at 9:18. A1 N2

[2]

(b) (i) $p = \dfrac{91-1}{2}$ (M1) for valid approach

$p = 45$ A1 N2

(ii) The period is 36 minutes. (A1) for correct value

$\therefore q = \dfrac{360^\circ}{36}$

$q = 10^\circ$ A1 N2

(iii) $r = \dfrac{1+91}{2}$ (M1) for valid approach

$r = 46$ A1 N2

[6]

(c) $-45\cos(10^\circ t) + 46 = 60$ (M1) for setting equation

$-45\cos(10^\circ t) - 14 = 0$ (A1) for correct equation

By considering the graph of the function

$y = -45\cos(10^\circ t) - 14$, $t = 82.81262$. (M1) for valid approach

Thus, the time is 10:22. A1 N4

[4]

4. (a) (i) $p = \dfrac{73-3}{2}$ (M1) for valid approach

$p = 35$ A1 N2

(ii) $q = \dfrac{360^\circ}{26}$ (M1)(A1) for correct value

$q = \left(\dfrac{180}{13}\right)^\circ$ A1 N3

(iii) $r = \dfrac{73+3}{2}$ (M1) for valid approach

$r = 38$ A1 N2

[7]

(b) The height

$= h(56)$

$= -35\cos\left(\left(\dfrac{180}{13}\right)^\circ \cdot 56\right) + 38$ (M1) for substitution

$= 18.11773386$

$= 18.1\,\text{m}$ A1 N2

[2]

(c) $h(t) = 10$

$-35\cos\left(\left(\dfrac{180}{13}\right)^\circ \cdot t\right) + 38 = 10$ (M1) for setting equation

$-35\cos\left(\left(\dfrac{180}{13}\right)^\circ \cdot t\right) + 28 = 0$ (A1) for correct equation

By considering the graph of the function

$y = -35\cos\left(\left(\dfrac{180}{13}\right)^\circ \cdot t\right) + 28,\ t = 75.337174.$ (M1) for valid approach

Thus, the time is 14:00. A1 N4

[4]

Chapter 13 Solution

Exercise 41

1. (a)

$$\frac{AB}{\sin 48^\circ} = \frac{10}{\sin 114^\circ}$$ (M1)(A1) for substitution

$$AB = \frac{10\sin 48^\circ}{\sin 114^\circ}$$

$$AB = 8.134732862$$

$AB = 8.13$ cm A1 N3

[3]

(b) $B\hat{A}C = 180^\circ - 114^\circ - 48^\circ$

$B\hat{A}C = 18^\circ$ (A1) for correct value

$$\frac{BC}{\sin 18^\circ} = \frac{10}{\sin 114^\circ}$$ (M1) for sine rule

$$BC = \frac{10\sin 18^\circ}{\sin 114^\circ}$$

$$BC = 3.382612127$$

$BC = 3.38$ cm A1 N3

[3]

2. (a) $B\hat{C}A = 180^\circ - 43^\circ - 92^\circ$ (M1) for valid approach

$B\hat{C}A = 45^\circ$

$$\frac{AB}{\sin 45^\circ} = \frac{21}{\sin 43^\circ}$$ (M1) for sine rule

$$AB = \frac{21\sin 45^\circ}{\sin 43^\circ}$$

$$AB = 21.77313506$$

$AB = 21.8$ cm A1 N3

[3]

(b) The area of ΔABC

$$= \frac{1}{2}(AC)(AB)\sin B\hat{A}C$$ (M1) for valid approach

$$= \frac{1}{2}(21)(21.77313506)\sin 92^\circ$$ (A1) for substitution

$$= 228.4786503$$

$= 228\ cm^2$ A1 N3

[3]

3. (a) $\frac{1}{2}(86)(BC)\sin 40^\circ = 1900$ (M1)(A1) for substitution

$BC = 68.74128537$

$BC = 68.7 \text{ cm}$ A1 N3

[3]

(b) $AB = \sqrt{86^2 + 68.74128537^2 - 2(86)(68.74128537)\cos 40^\circ}$ (M1)(A1) for substitution

$AB = 55.35374433$

$AB = 55.4 \text{ cm}$ A1 N3

[3]

4. (a) $\frac{1}{2}(35)(54)\sin B\hat{A}C = 892$ (M1)(A1) for substitution

$B\hat{A}C = 70.7198401^\circ$

$B\hat{A}C = 70.7^\circ$ A1 N3

[3]

(b) $BC = \sqrt{35^2 + 54^2 - 2(35)(54)\cos 70.7198401^\circ}$ (M1)(A1) for substitution

$BC = 53.78560244$

$BC = 53.8 \text{ cm}$ A1 N3

[3]

Exercise 42

1. (a) $A\hat{P}B = 180^\circ - 45^\circ - 34^\circ$ (M1) for valid approach

$A\hat{P}B = 101^\circ$ A1 N2

[2]

(b) $\frac{PB}{\sin 45^\circ} = \frac{120}{\sin 101^\circ}$ (M1)(A1) for substitution

$PB = 86.44097797$

$PB = 86.4 \text{ m}$ A1 N3

[3]

(c) $\sin 34^\circ = \frac{PC}{86.44097797}$ (M1) for valid approach

$PC = 48.33718145$

$PC = 48.3 \text{ m}$ A1 N2

[2]

2. (a) $P\hat{R}Q = 69^\circ - 24^\circ$ (M1) for valid approach

$P\hat{R}Q = 45^\circ$ A1 N2

[2]

(b) $Q\hat{P}R = 180^\circ - 69^\circ - 90^\circ$ (M1) for valid approach

$Q\hat{P}R = 21^\circ$ A1 N2

[2]

(c) $\frac{PQ}{\sin 45^\circ} = \frac{5}{\sin 21^\circ}$ (M1)(A1) for substitution

$PQ = 9.865653194$

$PQ = 9.87 \text{ m}$ A1 N3

[3]

3. (a) $\cos \hat{RPQ} = \dfrac{8^2 + 6^2 - 9^2}{2(8)(6)}$ (M1)(A1) for substitution

$\hat{RPQ} = 78.58484226$

Thus, the angle of depression of R from P is 78.6°. A1 N3

[3]

(b) $\cos 78.58484226^\circ = \dfrac{x}{8}$ (M1) for valid approach

$x = 1.583333333$

$x = 1.58$ A1 N2

[2]

(c) $\sin 78.58484226^\circ = \dfrac{h}{8}$ (M1) for valid approach

$h = 7.841750797$

$h = 7.84$ A1 N2

[2]

4. (a) 29° A1 N1

[1]

(b) $\dfrac{\sin \hat{BFA}}{10} = \dfrac{\sin 29^\circ}{30}$ (M1)(A1) for substitution

$\sin \hat{BFA} = 0.161603206$

$\hat{BFA} = 9.299964331^\circ$ (A1) for correct value

The angle of elevation of F from B

$= \hat{OBF}$

$= 9.299964331^\circ + 29^\circ$

$= 38.299964331^\circ$

$= 38.3^\circ$ A1 N4

[4]

(c) $\sin 38.299964331^\circ = \dfrac{OF}{30}$ (M1) for valid approach

$OF = 18.5933563$

Thus, the height of the vertical flagpole is 18.6 m. A1 N2

[2]

Exercise 43

1. (a) 60° A1 N1

[1]

(b) $\frac{1}{2}(3)(\text{BC})\sin 120^\circ = \frac{27\sqrt{3}}{4}$ (M1)(A1) for substitution

$\text{BC} = 9$ cm A1 N3

[3]

(c) The arc length of the sector BDC

$= 2\pi(9) \times \frac{60^\circ}{360^\circ}$ (A1) for substitution

$= 3\pi$ cm A1 N2

[2]

2. (a) $\cos \text{A}\hat{\text{B}}\text{C} = \frac{10^2 + 10^2 - (10\sqrt{3})^2}{2(10)(10)}$ (M1)(A1) for substitution

$\text{A}\hat{\text{B}}\text{C} = 120^\circ$ A1 N3

[3]

(b) 60° A1 N1

[1]

(c) The perimeter of the figure ADC

$= 10\sqrt{3} + 2(10) + 2\pi(10) \times \frac{60^\circ}{360^\circ}$ (M1)(A1) for substitution

$= 47.79248359$

$= 47.8$ cm A1 N3

[3]

3. (a) $\text{AC}^2 = 6^2 + 16^2 - 2 \times 6 \times 16 \times \cos 60^\circ$ M1A1

$\text{AC}^2 = 196$ A1

$\text{AC} = \sqrt{196}$

$\text{AC} = 14$ cm AG N0

[3]

(b) The area of this shape

$= \frac{1}{2}(6)(16)\sin 60^\circ + \pi\left(\frac{14}{2}\right)^2 \times \frac{180^\circ}{360^\circ}$ (M1)(A1) for substitution

$= 118.5382394$

$= 119\ \text{cm}^2$ A1 N3

[3]

4. (a) $\frac{1}{2}(8)(\text{AC})\sin 60^\circ = 24\sqrt{3}$ M1A1

$\text{AC} = 12 \text{ cm}$ AG N0

[2]

(b) $\text{AB} = \sqrt{12^2 + 8^2 - 2(12)(8)\cos\frac{\pi}{3}}$ (A1) for substitution

$\text{AB} = \sqrt{112} \text{ cm}$ A1 N2

[2]

(c) The perimeter of this shape

$= \sqrt{112} + 8 + 2\pi\left(\frac{12}{2}\right) \times \frac{180^\circ}{360^\circ}$ (M1)(A1) for substitution

$= 37.43256117$

$= 37.4 \text{ cm}$ A1 N3

[3]

Exercise 44

1. (a) The length of arc ABC

$= 2\pi(55) - 2\pi(55) \times \frac{155^\circ}{360^\circ}$ (M1)(A1) for substitution

$= 196.7858732$

$= 197 \text{ cm}$ A1 N3

[3]

(b) The perimeter of sector OABC

$= 196.7858732 + 55 + 55$ (M1) for valid approach

$= 306.7858732$

$= 307 \text{ cm}$ A1 N2

[2]

(c) The area of sector OABC

$= \pi(55)^2 - \pi(55)^2 \times \frac{155^\circ}{360^\circ}$ (M1) for valid approach

$= 5411.611512$

$= 5410 \text{ cm}^2$ A1 N2

[2]

2. (a) The length of arc ABC

$= 2\pi(20) \times \frac{54.5^\circ}{360^\circ}$ (A1) for substitution

$= 19.02408885 \text{ cm}$ (A1) for correct value

The perimeter of sector OABC

$= 19.02408885 + 20 + 20$ (M1) for valid approach

$= 59.02408885$

$= 59.0 \text{ cm}$ A1 N4

[4]

(b) The area of sector OABC

$= \pi(20)^2 \times \frac{54.5^\circ}{360^\circ}$ (A1) for substitution

$= 190.2408885$

$= 190 \text{ cm}^2$ A1 N2

[2]

3. (a) $2\pi(8.6)\times\frac{\theta^\circ}{360^\circ}=9.46$ (A1) for correct equation

$\theta=63.02535746$

$\theta=63.0$ A1 N2

[2]

(b) The reflex $A\hat{O}C$

$=360^\circ-63.02535746^\circ$ (M1) for valid approach

$=296.9746425^\circ$ (A1) for correct value

The area of sector $OADC$

$=\pi(8.6)^2\times\frac{296.9746425^\circ}{360^\circ}$ (A1) for substitution

$=191.6741927$

$=192\text{ cm}^2$ A1 N4

[4]

4. (a) $\pi(OC)^2\times\frac{114.5^\circ}{360^\circ}=14$ (A1) for correct equation

$OC^2=14.01119499$

$OC=3.743153081$

$OC=3.74\text{ cm}$ A1 N2

[2]

(b) The reflex $A\hat{O}C$

$=360^\circ-114.5^\circ$ (M1) for valid approach

$=245.5^\circ$ (A1) for correct value

The area of sector $OADC$

$=\pi(3.743153081)^2\times\frac{245.5^\circ}{360^\circ}$ (A1) for substitution

$=30.01746725$

$=30.0\text{ cm}^2$ A1 N4

[4]

Exercise 45

1. (a) The required area

$= \pi(125)^2 \times \frac{142^\circ}{360^\circ}$ (A1) for substitution

$= 19362.24639$

$= 19400 \text{ cm}^2$ A1 N2

[2]

(b) The required area

$= \frac{1}{2}(125)(125)\sin 142^\circ$ (A1) for substitution

$= 4809.855276$

$= 4810 \text{ cm}^2$ A1 N2

[2]

(c) The required area

$= 19362.24639 - 4809.855276$ (M1) for valid approach

$= 14552.39111$

$= 14600 \text{ cm}^2$ A1 N2

[2]

2. (a) The required length

$= 2\pi(1740) \times \frac{80^\circ}{360^\circ}$ (A1) for substitution

$= 2429.498319$

$= 2430 \text{ cm}$ A1 N2

[2]

(b) $AB = \sqrt{1740^2 + 1740^2 - 2(1740)(1740)\cos 80^\circ}$ (M1)(A1) for substitution

$AB = 2236.900882$

$AB = 2240 \text{ cm}$ A1 N3

[3]

(c) The required perimeter

$= 2429.498319 + 2236.900882$ (M1) for correct approach

$= 4666.399201$

$= 4670 \text{ cm}$ A1 N2

[2]

3. (a) $\cos A\hat{O}B = \dfrac{20^2 + 20^2 - 32^2}{2(20)(20)}$ (M1)(A1) for substitution

$\cos A\hat{O}B = -0.28$

$A\hat{O}B = 106.2602047^\circ$

$A\hat{O}B = 106^\circ$ A1 N3

[3]

(b) The area of the sector AOB

$= \pi(20)^2 \times \dfrac{106.2602047^\circ}{360^\circ}$ (A1) for substitution

$= 370.9180872$

$= 371\,\text{cm}^2$ A1 N2

[2]

(c) The area of the shaded region

$= 370.9180872 - 192$ (M1) for valid approach

$= 178.9180872$

$= 179\,\text{cm}^2$ A1 N2

[2]

4. (a) $\cos A\hat{O}B = \dfrac{40^2 + 40^2 - 60^2}{2(40)(40)}$ (M1)(A1) for substitution

$\cos A\hat{O}B = -0.125$

$A\hat{O}B = 97.18075578^\circ$

$A\hat{O}B = 97.2^\circ$ A1 N3

[3]

(b) The length of the minor arc AB

$= 2\pi(40) \times \dfrac{97.18075578^\circ}{360^\circ}$ (A1) for substitution

$= 67.84496632$

$= 67.8\,\text{cm}^2$ A1 N2

[2]

(c) The perimeter of the shaded segment

$= (2\pi(40) - 67.84496632) + 60$ (M1) for valid approach

$= 243.482446$

$= 243\ \text{cm}$ A1 N2

[2]

Exercise 46

1. (a) The bearing of C from E

$= 360^\circ - (180^\circ - 77^\circ)$ (M1) for valid approach

$= 257^\circ$ A1 N2

[2]

(b) (i) $A\hat{C}E = 180^\circ - 77^\circ$

$A\hat{C}E = 103^\circ$ (A1) for correct value

$A\hat{E}C = 180^\circ - 103^\circ - 51^\circ$

$A\hat{E}C = 26^\circ$ A1 N2

(ii) $\dfrac{AE}{\sin 103^\circ} = \dfrac{800}{\sin 26^\circ}$ (M1)(A1) for substitution

$AE = 1778.164593$

$AE = 1780 \text{ km}$ A1 N3

[5]

(c) $DE = \sqrt{1778.164593^2 + 1350^2 - 2(1778.164593)(1350)\cos 51^\circ}$ (M1)(A1) for substitution

$DE = 1401.061804$

$DE = 1400 \text{ km}$ A1 N3

[3]

(d) (i) B lies on AC such that $BE \perp AC$. (M1) for valid approach

$\sin B\hat{A}E = \dfrac{BE}{AE}$

$BE = 1381.893432$

$BE = 1380 \text{ km}$ A1 N2

(ii) The time required

$= \dfrac{DE}{62}$ (M1) for valid approach

$= \dfrac{1401.061804}{62}$

$= 22.59777103 \text{ h}$

The speed of the boat

$= \dfrac{BE}{22.59777103}$ (M1) for valid approach

$= \dfrac{1381.893432}{22.59777103}$

$= 61.15175829$

$= 61.2 \text{ km/h}$ A1 N3

[5]

2. (a) $A\hat{D}C = 160^\circ - 90^\circ$

$A\hat{D}C = 70^\circ$ (A1) for correct value

$\frac{AC}{\sin 70^\circ} = \frac{15}{\sin 58^\circ}$ (M1) for sine rule

$AC = 16.62097866$

$AC = 16.6 \text{ km}$ A1 N3

[3]

(b) $D\hat{A}C = 180^\circ - 70^\circ - 58^\circ$

$D\hat{A}C = 52^\circ$ (A1) for correct value

The area of the triangle DAC

$= \frac{1}{2}(15)(16.62097866)\sin 52^\circ$ (A1) for substitution

$= 98.2313244$

$= 98.2 \text{ km}^2$ A1 N3

[3]

(c) (i) The area of the triangle ABC

$= 2(98.2313244)$

$= 196.4626488$

$= 196 \text{ km}^2$ A1 N1

(ii) $\frac{1}{2}(16.620979)(BC)\sin 56^\circ = 196.4626488$ (M1)(A1) for correct equation

$BC = 28.51538144$

$BC = 28.5 \text{ km}$ A1 N3

[4]

(d) (i) $\frac{DC}{\sin 52^\circ} = \frac{15}{\sin 58^\circ}$ (A1) for substitution

$DC = 13.93807893$

$DC = 13.9 \text{ km}$ A1 N2

(ii) $BD = \sqrt{13.93807893^2 + 28.51538144^2 - 2(13.93807893)(28.51538144)\cos(58^\circ + 56^\circ)}$ (A1) for substitution

$BD = 36.47892111$

$BD = 36.5 \text{ km}$ A1 N2

(iii) $\frac{28.51538144}{1} = \frac{36.47892111}{T}$ (M1) for valid approach

$T = 1.279271722$

Thus, the time taken is 1.28 hours. A1 N2

[6]

3. (a) $A\hat{B}C = 360^\circ - 312^\circ$

$A\hat{B}C = 48^\circ$ (A1) for correct value

$\frac{AC}{\sin 48^\circ} = \frac{60}{\sin 83^\circ}$ (A1) for substitution

$AC = 44.9235428$

$AC = 44.9 \text{ km}$ A1 N3

[3]

(b) The area of the triangle ABC

$= \frac{1}{2}(44.9235428)(60)\sin 49^\circ$ (M1)(A1) for substitution

$= 1017.126844$

$= 1020 \text{ km}^2$ A1 N3

[3]

(c) The area of the triangle ACD

$= 1.5(1017.126844)$

$= 1525.690266$ (A1) for correct value

$\frac{1}{2}(DC)(AC)\sin\theta^\circ = 1525.690266$ (M1) for setting equation

$\frac{1}{2}(83)(44.9235428)\sin\theta^\circ = 1525.690266$ (A1) for correct equation

$\sin\theta^\circ = 0.8183597859$

$\theta^\circ = 54.92093749^\circ$

$\theta^\circ = 54.9^\circ$ A1 N4

[4]

(d) $\frac{BC}{\sin(180^\circ - 48^\circ - 83^\circ)} = \frac{60}{\sin 83^\circ}$

$BC = 45.62263905$ (A1) for correct value

$BD = \sqrt{\begin{matrix}83^2 + 45.62263905^2 \\ -2(83)(45.62263905) \\ \times\cos(54.92093749^\circ + 83^\circ)\end{matrix}}$ (M1)(A1) for substitution

$BD = 120.7954013$

$\frac{BD}{\text{Speed of Q}} = \frac{BC + DC}{50}$ (M1) for valid approach

$\frac{120.7954013}{\text{Speed of Q}} = \frac{45.62263905 + 83}{50}$

$\text{Speed of Q} = 46.95728613$

$\text{Speed of Q} = 47.0 \text{ km/h}$ A1 N5

[5]

4. (a) $\hat{BAD} = 220^\circ - 180^\circ$

$\hat{BAD} = 40^\circ$ (A1) for correct value

$\hat{BDA} = 180^\circ - 40^\circ - 61^\circ$

$\hat{BDA} = 79^\circ$ (A1) for correct value

$\frac{BD}{\sin 40^\circ} = \frac{80}{\sin 79^\circ}$ (A1) for substitution

$BD = 52.38547754$

$BD = 52.4 \text{ km}$ A1 N4

[4]

(b) The area of the triangle ABD

$= \frac{1}{2}(52.38547754)(80)\sin 61^\circ$ (M1)(A1) for substitution

$= 1832.694841$

$= 1830 \text{ km}^2$ A1 N3

[3]

(c) The area of the triangle BCD is 1832.694841 km^2.

$\frac{1}{2}(CD)(BD)\sin\theta^\circ = 1832.694841$ (M1) for setting equation

$\frac{1}{2}(72)(52.38547754)\sin\theta^\circ = 1832.694841$ (A1) for substitution

$\sin\theta^\circ = 0.9717996746$ (A1) for correct value

$\theta^\circ = 180^\circ - 76.36074617^\circ$

$\theta^\circ = 103.6392538^\circ$

$\theta^\circ = 104^\circ$ A1 N4

[4]

(d) $BC = \sqrt{72^2 + 52.38547754^2 - 2(72)(52.38547754)\cos 103.6392538^\circ}$ (A1) for substitution

$BC = 98.52440125$ (A1) for correct value

The total distance

$= 98.52440125 + 80$

$= 178.52440125$

The minimum time required

$= \frac{178.52440125}{70}$ (M1) for valid approach

$= 2.550348589 \text{ h}$ (A1) for correct value

$= 2 \text{ hours } 33 \text{ minutes}$ A1 N5

[5]

Exercise 47

1. (a) $\dfrac{\sin A\hat{C}B}{20.8} = \dfrac{\sin 71.5^\circ}{26.6}$ (M1)(A1) for substitution

$A\hat{C}B = 47.86330515^\circ$

$A\hat{C}B = 47.9^\circ$ A1 N3

[3]

(b) (i) $B\hat{A}C = 180^\circ - 71.5^\circ - 47.86330515^\circ$ (M1) for valid approach

$B\hat{A}C = 60.63669485^\circ$

$B\hat{A}C = 60.6^\circ$ A1 N2

(ii) $\dfrac{BC}{\sin 60.63669485^\circ} = \dfrac{26.6}{\sin 71.5^\circ}$ (M1)(A1) for substitution

$BC = 24.44592146$

$BC = 24.4 \text{ km}$ A1 N3

[5]

(c) (i) $\cos B\hat{O}C = \dfrac{14^2 + 14^2 - 24.44592146^2}{2(14)(14)}$ (M1)(A1) for substitution

$B\hat{O}C = 121.634431^\circ$ (A1) for correct value

Reflex $B\hat{O}C$

$= 360^\circ - 121.634431^\circ$

$= 238.365569^\circ$

$= 238^\circ$ A1 N4

(ii) The required area

$= \pi(14)^2 \times \dfrac{238.365569^\circ}{360^\circ}$ (A1) for substitution

$= 407.7058723$

$= 408 \text{ cm}^2$ A1 N2

[6]

2. (a) $\dfrac{AC}{\sin 50^\circ} = \dfrac{43.2}{\sin 70^\circ}$ (M1)(A1) for substitution

$AC = 35.21696266$

$AC = 35.2 \text{ cm}$ A1 N3

[3]

(b) $B\hat{A}C = 180^\circ - 50^\circ - 70^\circ$ (M1) for valid approach

$B\hat{A}C = 60^\circ$ (A1) for correct value

$\dfrac{BC}{\sin 60^\circ} = \dfrac{43.2}{\sin 70^\circ}$ (M1)(A1) for substitution

$BC = 39.81333536$

$BC = 39.8 \text{ cm}$ A1 N5

[5]

(c) (i) $\cos B\hat{O}C = \dfrac{23^2 + 23^2 - 39.81333536^2}{2(23)(23)}$ (M1)(A1) for substitution

$B\hat{O}C = 119.8813635^\circ$

$B\hat{O}C = 120^\circ$ A1 N3

(ii) The area of the sector OBDC

$= \pi(23)^2 \times \dfrac{119.8813635^\circ}{360^\circ}$ (A1) for substitution

$= 553.4198315$

$= 553 \text{ cm}^2$ A1 N2

(iii) The required area

$= 553.4198315 - \dfrac{1}{2}(23)(23)\sin 119.8813635^\circ$ (M1)(A1) for substitution

$= 324.0827669$

$= 324 \text{ cm}^2$ A1 N3

[8]

3. (a) $\dfrac{AC}{\sin 40^\circ} = \dfrac{11}{\sin 21^\circ}$ (M1)(A1) for substitution

$AC = 19.73017876$

$AC = 19.7 \text{ cm}$ A1 N3

[3]

(b) $O\hat{A}C = 180^\circ - 40^\circ - 21^\circ$ (M1) for valid approach

$O\hat{A}C = 119^\circ$

$\dfrac{OC}{\sin 119^\circ} = \dfrac{11}{\sin 21^\circ}$ (M1)(A1) for substitution

$OC = 26.84619758$

$OC = 26.8 \text{ cm}$ A1 N4

[4]

(c) The area of the sector OBA

$= \pi(11)^2 \times \dfrac{40^\circ}{360^\circ}$ (A1) for substitution

$= 42.2369679$ (A1) for correct value

The area of the triangle OAC

$= \dfrac{1}{2}(11)(26.84619758)\sin 40^\circ$ (A1) for substitution

$= 94.91021744$ (A1) for correct value

The required area

$= 94.91021744 - 42.2369679$ (M1) for valid approach

$= 52.67324954$

$= 52.7 \text{ cm}^2$ A1 N6

[6]

4. (a) $\dfrac{\sin A\hat{C}B}{28} = \dfrac{\sin 63^\circ}{47}$ (M1)(A1) for substitution

$\sin A\hat{C}B = 0.530812397$

$A\hat{C}B = 32.06036169^\circ$

$A\hat{C}B = 32.1^\circ$ A1 N3

[3]

(b) $O\hat{A}C = 180^\circ - 63^\circ - 32.06036169^\circ$ (M1) for valid approach

$O\hat{A}C = 84.93963831^\circ$

$\dfrac{OC}{\sin 84.93963831^\circ} = \dfrac{47}{\sin 63^\circ}$ (M1)(A1) for substitution

$OC = 52.54373345$

$OC = 52.5 \text{ cm}$ A1 N4

[4]

(c) The length of the arc ADB

$= 2\pi(28) \times \dfrac{63^\circ}{360^\circ}$ (A1) for substitution

$= 30.78760801$ (A1) for correct value

The required perimeter

$= 30.78760801 + (52.54373345 - 28) + 47$ (M1) for valid approach

$= 102.3313415$

$= 102 \text{ cm}$ A1 N4

[4]

Chapter 14 Solution

Exercise 48

1. (a) $\pi r^2 = 9\pi$ (M1) for setting equation

$r^2 = 9$

$r = 3$ cm A1 N2

[2]

(b) 12π cm^3 A1 N1

[1]

(c) The slant height l of the circular cone

$= \sqrt{3^2 + 4^2}$ (M1) for valid approach

$= 5$

The total surface area

$= \pi r^2 + \pi rl$ (M1) for valid approach

$= \pi(3)^2 + \pi(3)(5)$ (A1) for substitution

$= 75.39822369$

$= 75.4$ cm^2 A1 N4

[4]

2. (a) $\pi r^2 = 37$ (M1) for setting equation

$r = \sqrt{\frac{37}{\pi}}$

$r = 3.431831259$

$r = 3.43$ cm A1 N2

[2]

(b) 84.7 cm^3 A2 N2

[2]

(c) The total surface area

$= 2\pi r^2 + \pi r^2$ (M1) for valid approach

$= 3\pi r^2$

$= 3(37)$ (A1) for substitution

$= 111$ cm^2 A1 N3

[3]

3. (a) $V = \frac{1}{3}\pi r^2 h$ (M1) for setting equation

$150 = \frac{1}{3}\pi r^2 (13)$

$r = \sqrt{\frac{450}{13\pi}}$

$r = 3.319400418$

$r = 3.32$ cm A1 N2

[2]

(b) $l = \sqrt{3.319400418^2 + 13^2}$ (M1) for valid approach

$l = 13.41709429$

$l = 13.4$ cm A1 N2

[2]

(c) The curved surface area

$= \pi rl$ (M1) for valid approach

$= \pi(3.319400418)(13.41709429)$

$= 139.9161959$

$= 140\ \text{cm}^2$ A1 N2

[2]

4. (a) $A = \pi rl$ (M1) for setting equation

$369\pi = \pi r(41)$

$r = 9$ cm A1 N2

[2]

(b) The vertical height h

$= \sqrt{41^2 - 9^2}$ (M1) for valid approach

$= 40$ cm A1 N2

[2]

(c) The volume

$= \frac{1}{3}\pi r^2 h$ (M1) for valid approach

$= \frac{1}{3}\pi(9)^2(40)$

$= 1080\pi\ \text{cm}^3$ A1 N2

[2]

Exercise 49

1. (a) The volume

$= \frac{1}{3}\pi R^2 H + \pi r^2 h$ (M2) for valid approach

$= \frac{1}{3}\pi(12)^2(16) + \pi(12)^2(5)$ (A1) for substitution

$= 4674.689869$

$= 4670\ \text{m}^3$ A1 N4

[4]

(b) The slant height of the top

$= \sqrt{12^2 + 16^2}$ (M1) for valid approach

$= 20$

The area

$= \pi rl$ (M1) for valid approach

$= \pi(12)(20)$

$= 753.9822369$

$= 754\ \text{m}^2$ A1 N3

[3]

2. (a) The volume

$= \frac{1}{3}\pi r^2 h + \frac{2}{3}\pi r^3$ (M2) for valid approach

$= \frac{1}{3}\pi(8)^2(6) + \frac{2}{3}\pi(8)^3$ (A1) for substitution

$= 1474.454152$

$= 1470\ \text{cm}^3$ A1 N4

[4]

(b) The slant height of the circular cone

$= \sqrt{6^2 + 8^2}$ (M1) for valid approach

$= 10$

The total surface area

$= \pi rl + 2\pi r^2$ (M1) for valid approach

$= \pi(8)(10) + 2\pi(8)^2$

$= 653.4512719$

$= 653\ \text{cm}^2$ A1 N3

[3]

3. (a) $V = \frac{2}{3}\pi r^3 + \pi r^2 h$ (M2) for valid approach

$54000\pi = \frac{2}{3}\pi r^3 + \pi r^2(40)$ (A1) for substitution

$54000 = \frac{2}{3}r^3 + 40r^2$

$\frac{2}{3}r^3 + 40r^2 - 54000 = 0$ (M1) for setting equation

$r = 30$ m A1 N5

[5]

(b) The area

$= 2\pi r^2$ (M1) for valid approach

$= 2\pi(30)^2$

$= 5654.866776$

$= 5650 \text{ m}^2$ A1 N2

[2]

4. (a) $A = 2\pi r^2 + 2\pi rh + 2\pi r^2$ (M2) for setting equation

$28\pi = 4\pi r^2 + 2\pi r(3)$ (A1) for substitution

$28 = 4r^2 + 6r$

$2r^2 + 3r - 14 = 0$ (M1) for quadratic equation

$(2r + 7)(r - 2) = 0$

$2r + 7 = 0$ or $r - 2 = 0$

$r = -\frac{7}{2}$ (*Rejected*) or $r = 2$ mm A1 N5

[5]

(b) The volume

$= \frac{4}{3}\pi r^3 + \pi r^2 h$ (M1) for valid approach

$= \frac{4}{3}\pi(2)^3 + \pi(2)^2(3)$

$= 71.20943348$

$= 71.2 \text{ mm}^3$ A1 N2

[2]

Exercise 50

1. (a) The volume

$= \frac{2}{3}\pi r^3$ (M1) for valid approach

$= \frac{2}{3}\pi(22)^3$

$= 22301.11905$ (A1) for correct value

$= 22300$

$= 2.23\times10^4 \text{ cm}^3$ A1 N3

[3]

(b) $V = \pi r^2 h$ (M1) for setting equation

$22301.11905 = \pi r^2(26)$ (A1) for substitution

$r^2 = 273.025641$

$r = 16.52348756$

$r = 16.5 \text{ cm}$ A1 N3

[3]

2. (a) The volume

$= \frac{1}{3}Ah$ (M1) for valid approach

$= \frac{1}{3}(8\pi)^2(35)$

$= 7369.304619$ (A1) for correct value

$= 7370$

$= 7.37\times10^3 \text{ cm}^3$ A1 N3

[3]

(b) $V = \frac{4}{3}\pi r^3$ (M1) for setting equation

$7369.304619 = \frac{4}{3}\pi r^3$ (A1) for substitution

$r^3 = 1759.291886$

$r = 12.07200203$

$r = 12.1 \text{ cm}$ A1 N3

[3]

3. (a) The volume

$= \pi r^2 h$ (M1) for valid approach

$= \pi(7)^2(100)$

$= 4900\pi \text{ cm}^3$ A1 N2

[2]

(b) $V = 10\left(\frac{2}{3}\pi r^3\right)$ (M1) for setting equation

$4900\pi = \frac{20}{3}\pi r^3$ (A1) for substitution

$r^3 = 735$

$r = 9.024623926$ (A1) for correct value

$r = 9.02$

$r = 9.02 \times 10^1 \text{ cm}$ A1 N4

[4]

4. (a) The volume

$= \frac{1}{3}\pi r^2 h$ (M1) for valid approach

$= \frac{1}{3}\pi(27)^2(27)$

$= 20611.9894$ (A1) for correct value

$= 20600$

$= 2.06 \times 10^4 \text{ cm}^3$ A1 N3

[3]

(b) $V = 27\left(\frac{4}{3}\pi r^3\right)$ (M1) for setting equation

$4(20611.9894) = 36\pi r^3$ (A1) for substitution

$r^3 = 729$

$r = 9$ A1

The ratio

$= 27 : 9$

$= 3 : 1$ A1 N4

[4]

Exercise 51

1. (a) The total surface area

$= 4\pi r^2$

$= 4\pi(15)^2$ (A1) for substitution

$= 2827.433388$

$= 2830\,\text{cm}^2$ A1 N2

[2]

(b) $V = 4\pi r^2$

$2827.433388 \times (1+30\%) = 4\pi r^2$ (M2) for setting equation

$r^2 = 292.5$ (M1) for valid approach

$r = 17.10263138$

$r = 17.1\,\text{cm}$ A1 N4

[4]

2. (a) The total surface area

$= 4\pi r^2$

$= 4\pi(14)^2$ (A1) for substitution

$= 2463.00864$

$= 2460\,\text{cm}^2$ A1 N2

[2]

(b) $V = 4\pi r^2$

$2463.00864 \times (1+15\%) = 4\pi r^2$ (M2) for setting equation

$r^2 = 225.4$ (M1) for valid approach

$r = 15.01332741$

The percentage increase

$$= \frac{\frac{4}{3}\pi(15.01332741)^3 - \frac{4}{3}\pi(14)^3}{\frac{4}{3}\pi(14)^3} \times 100\%$$ M1

$= 23.32376089\%$

$= 23.3\%$ A1 N5

[5]

3. (a) The total surface area

$= 2\pi r^2 + 2\pi rh$

$= 2\pi(18)^2 + 2\pi(18)(8)$ (A1) for substitution

$= 2940.530724$

$= 2940\ \text{cm}^2$ A1 N2

[2]

(b) Increase in total surface area

$= 2(2rh)$ (M1) for valid approach

$= 2(2)(18)(8)$

$= 576$ (A1) for correct value

The percentage increase

$= \dfrac{576}{2940.530724} \times 100\%$ M1

$= 19.58830069\%$

$= 19.6\%$ A1 N4

[4]

4. (a) The total surface area

$= \pi r^2 + \pi rl$

$= \pi(7)^2 + \pi(7)(25)$ (A1) for substitution

$= 703.7167544$

$= 704\ \text{cm}^2$ A1 N2

[2]

(b) The vertical height h

$= \sqrt{25^2 - 7^2}$ (M1) for valid approach

$= 24$

Increase in total surface area

$= 2\left(\dfrac{1}{2}(2r)(h)\right)$ (M1) for valid approach

$= 2rh$

$= 2(7)(24)$

$= 336$ (A1) for correct value

The percentage increase

$= \dfrac{336}{703.7167544} \times 100\%$ M1

$= 47.74648293\%$

$= 47.7\%$ A1 N5

[5]

Exercise 52

1. (a) The volume

$= \frac{2}{3}\pi r^3$ (M1) for valid approach

$= \frac{2}{3}\pi(10)^3$ (A1) for substitution

$= 2094.395102$

$= 2090\ \text{cm}^3$ A1 N3

[3]

(b) The total surface area

$= 2\pi r^2 + \pi r^2$ (M1) for valid approach

$= 3\pi r^2$

$= 3\pi(10)^2$ (A1) for substitution

$= 942.4777961$

$= 942\ \text{cm}^2$ A1 N3

[3]

(c) $V = \frac{1}{3}\pi r^2 h$ (M1) for setting equation

$4(2094.395102) = \frac{1}{3}\pi\left(\frac{38}{2}\right)^2 (\text{OV})$ (A1) for substitution

$\text{OV} = 22.16066482$

$\text{OV} = 22.2\ \text{cm}$ A1 N3

[3]

(d) The slant height l

$= \sqrt{22.16066482^2 + 19^2}$ (M1) for valid approach

$= 29.19066743$

$\cos \text{A}\hat{\text{V}}\text{B} = \frac{\text{VA}^2 + \text{VB}^2 - \text{AB}^2}{2(\text{VA})(\text{VB})}$ (M1) for cosine rule

$\cos \text{A}\hat{\text{V}}\text{B} = \frac{29.19066743^2 + 29.19066743^2 - 38^2}{2(29.19066743)(29.19066743)}$ (A1) for substitution

$\text{A}\hat{\text{V}}\text{B} = 81.21792258$

$\text{A}\hat{\text{V}}\text{B} = 81.2^\circ$ A1 N4

[4]

(e) The total surface area

$= \pi r^2 + \pi rl$ (M1) for valid approach

$= \pi(19)^2 + \pi(19)(29.19066743)$ (A1) for substitution

$= 2876.513489$

$= 2880\ \text{cm}^2$ A1 N3

[3]

2. (a) The volume

$= A_1 h_1$ (M1) for valid approach

$= (260)(100)$

$= 26000 \text{ cm}^3$ A1 N2

[2]

(b) The total surface area

$= 2A_1 + ph_1$ (M1) for valid approach

$= 2(260) + (62)(100)$ (A1) for substitution

$= 6720 \text{ cm}^2$ A1 N3

[3]

(c) $V = \frac{1}{3} A_2 h_2$ (M1) for setting equation

$26000 = \frac{1}{3}(\text{AD})^2(40)$ (A1) for substitution

$\text{AD} = 44.15880433$

$\text{AD} = 44.2 \text{ cm}$ A1 N3

[3]

(d) $\tan \hat{\text{VMO}} = \frac{\text{OV}}{\text{OM}}$ (M1) for tangent ratio

$\tan \hat{\text{VMO}} = \frac{\text{OV}}{\frac{1}{2}\text{AD}}$

$\tan \hat{\text{VMO}} = \frac{40}{\frac{1}{2}(44.15880433)}$ (A1) for substitution

$\hat{\text{VMO}} = 61.10195875$

$\hat{\text{VMO}} = 61.1^\circ$ A1 N3

[3]

(e) OA

$= \sqrt{OM^2 + AM^2}$ (M1) for valid approach

$= \sqrt{OM^2 + OM^2}$

$= \sqrt{2\left(\frac{1}{2}(44.15880433)\right)^2}$

$= 31.22498999$ (A1) for correct value

$\tan V\hat{A}O = \frac{OV}{OA}$ (M1) for tangent ratio

$\tan V\hat{A}O = \frac{40}{31.22498999}$ (A1) for substitution

$V\hat{A}O = 52.02352051$

$V\hat{A}O = 52.0^\circ$ A1 N5

[5]

3. (a) $\sin \hat{OVA} = \frac{OA}{VA}$ (M1) for sine ratio

$\sin \hat{OVA} = \frac{40}{104}$

$\hat{OVA} = 22.61986495$

$\hat{OVA} = 22.6^{\circ}$ A1 N2

[2]

(b) The total surface area

$= \pi r^2 + \pi rl$ (M1) for valid approach

$= \pi(40)^2 + \pi(40)(104)$ (A1) for substitution

$= 18095.57368$

$= 18100 \text{ cm}^2$ A1 N3

[3]

(c) The vertical height h

$= \sqrt{104^2 - 40^2}$ (M1) for valid approach

$= 96$

The volume

$= \frac{1}{3}\pi r^2 h$ (M1) for valid approach

$= \frac{1}{3}\pi(40)^2(96)$ (A1) for substitution

$= 160849.5439$

$= 161000 \text{ cm}^3$ A1 N4

[4]

(d) $\tan \hat{OVA} = \frac{R}{H}$ (M1) for tangent ratio

$\therefore \frac{40}{96} = \frac{R}{H}$

$R = \frac{5}{12}H$ A1 N2

[2]

(e) $V = \frac{1}{3}\pi R^2 H$ (M1) for setting equation

$\frac{160849.5439}{2} = \frac{1}{3}\pi\left(\frac{5}{12}H\right)^2 (H)$ (A1) for substitution

$80424.77193 = \frac{25}{432}\pi H^3$

$H^3 = 442368$ (M1) for valid approach

$H = 76.19525049$

$R = \frac{5}{12}(76.19525049)$ (M1) for substitution

$R = 31.74802104$

Thus, $H = 76.2$ cm and $R = 31.7$ cm. A2 N6

[6]

4. (a) $\cos \hat{OVA} = \frac{VO}{VA}$ (M1) for cosine ratio

$\cos \hat{OVA} = \frac{56}{70}$

$\hat{OVA} = 36.86989765$

$\hat{OVA} = 36.9^\circ$ A1 N2

[2]

(b) OA

$= \sqrt{70^2 - 56^2}$ (M1) for valid approach

$= 42$

$AD^2 = OA^2 + OD^2$ (M1) for setting equation

$AD^2 = OA^2 + OA^2$

$AD^2 = 42^2 + 42^2$ (A1) for substitution

$AD = 59.39696962$

$AD = 59.4 \text{ cm}$ A1 N4

[4]

(c) The volume

$= \frac{1}{3} A_1 h_1$ (M1) for valid approach

$= \frac{1}{3} (AD)^2 (VO)$

$= \frac{1}{3} (59.39696962)^2 (56)$ (A1) for substitution

$= 65856 \text{ cm}^3$ A1 N3

[3]

(d) $V = \frac{1}{3} A_2 h_2$ (M1) for setting equation

$\frac{65856}{2} = \frac{1}{3} (x)(56 \times 2^{-\frac{1}{3}})$ (A1) for substitution

$x = 2222.500732$ (A1) for correct value

y

$= AD^2$

$= 42^2 + 42^2$

$= 3528$ (A1) for correct value

$\therefore x : y$

$= 2222.500732 : 3528$ (M1) for valid approach

$= 1 : 0.630$ A1 N6

[6]

Chapter 15 Solution

Exercise 53

1. (a) $f'(x) = 3(4x^3) + \frac{1}{2}(1)$ (A1) for correct derivatives

$f'(x) = 12x^3 + \frac{1}{2}$ A1 N2

[2]

(b) The gradient

$= 12(0)^3 + \frac{1}{2}$ (A1) for substitution

$= \frac{1}{2}$ A1 N2

[2]

(c) The gradient of the normal

$= \frac{-1}{12(-1)^3 + \frac{1}{2}}$ (A1) for substitution

$= \frac{2}{23}$ A1 N2

[2]

2. (a) $f'(x) = 4(2x) - 0$ (A1) for correct derivatives

$f'(x) = 8x$ A1 N2

[2]

(b) The gradient

$= 8\left(\frac{1}{4}\right)$ (A1) for substitution

$= 2$ A1 N2

[2]

(c) The gradient of the normal is $\frac{-1}{8a}$. (M1) for valid approach

$\frac{-1}{8a} = \frac{1}{16}$ (A1) for correct equation

$-16 = 8a$

$a = -2$ A1 N3

[3]

3. (a) $f(-2) = \frac{1}{2}(-2)^2 - \frac{1}{-2}$ (A1) for substitution

$f(-2) = \frac{5}{2}$ A1 N2

[2]

(b) $f'(x) = \frac{1}{2}(2x) - (-x^{-2})$ (A1) for correct derivatives

$f'(x) = x + \frac{1}{x^2}$ A1 N2

[2]

(c) The gradient of the normal is $\frac{-1}{a + \frac{1}{a^2}}$. (M1) for valid approach

$\frac{-1}{a + \frac{1}{a^2}} = -\frac{9}{28}$ (A1) for correct equation

$28 = 9\left(a + \frac{1}{a^2}\right)$

$9a + \frac{9}{a^2} - 28 = 0$

By considering the graph of $y = 9a + \frac{9}{a^2} - 28$,

$a = -0.524461$ (*Rejected*),

$a = 0.6355726$ (*Rejected*) or $a = 3$. (A1) for correct values

$\therefore a = 3$ A1 N4

[4]

4. (a) 0 A1 N1

[1]

(b) $f'(x) = \frac{1}{a}(3x^2) + 1$ (A1) for correct derivatives

$f'(x) = \frac{3}{a}x^2 + 1$ A1 N2

[2]

(c) The gradient of the normal is $\frac{-1}{\frac{3}{a}(2)^2 + 1}$. (M1) for valid approach

$\frac{-1}{\frac{3}{a}(2)^2 + 1} = -\frac{1}{3}$ (A1) for correct equation

$\frac{3}{a}(4) + 1 = 3$

$\frac{12}{a} = 2$

$a = 6$ A1 N3

[3]

Exercise 54

1. (a) $f'(x) = 6(4x^3) - 21(2x)$ (A1) for correct derivatives

$f'(x) = 24x^3 - 42x$ A1 N2

[2]

(b) The gradient of L

$= 24(2)^3 - 42(2)$ (A1) for substitution

$= 108$ A1 N2

[2]

(c) The equation of L:

$y - 12 = 108(x - 2)$ (A1) for substitution

$y - 12 = 108x - 216$

$y = 108x - 204$ A1 N2

[2]

2. (a) $f'(x) = 3(1) - 4(-2x^{-3})$ (A1) for correct derivatives

$f'(x) = 3 + 8x^{-3}$ A1 N2

[2]

(b) The gradient of L

$= \dfrac{-1}{3 + 8(1)^{-3}}$ (A1) for substitution

$= -\dfrac{1}{11}$ A1 N2

[2]

(c) The equation of L:

$y - (-1) = -\dfrac{1}{11}(x - 1)$ (A1) for substitution

$y + 1 = -\dfrac{1}{11}x + \dfrac{1}{11}$

$y = -\dfrac{1}{11}x - \dfrac{10}{11}$ A1 N2

[2]

3. (a) $f'(x) = a(3x^2) - 2(2x) + 0$ (A1) for correct derivatives

$f'(x) = 3ax^2 - 4x$ A1 N2

[2]

(b) $3a(3)^2 - 4(3) = 96$ (A1) for substitution

$27a = 108$

$a = 4$ A1 N2

[2]

(c) The equation of L:

$y - (27(4) - 17) = 96(x - 3)$ (A1) for substitution

$y - 91 = 96x - 288$

$y = 96x - 197$ A1 N2

[2]

4. (a) $f'(x) = 0 - a(3x^2)$ (A1) for correct derivatives

$f'(x) = -3ax^2$ A1 N2

[2]

(b) The gradient of L

$= \dfrac{-1}{-3a(2)^2}$ (A1) for substitution

$= \dfrac{1}{12a}$ A1 N2

[2]

(c) -1 A1 N1

[1]

(d) The equation of L:

$y - 11 = -\dfrac{1}{12}(x - 2)$ (A1) for substitution

$y - 11 = -\dfrac{1}{12}x + \dfrac{1}{6}$

$y = -\dfrac{1}{12}x + \dfrac{67}{6}$ A1 N2

[2]

Exercise 55

1. (a) $f'(x) = 2(2x) - 1$ (A1) for correct derivatives

$f'(x) = 4x - 1$ A1 N2

[2]

(b) $g'(x) = 2x$ A1 N1

[1]

(c) $f'(x) = g'(x)$

$4x - 1 = 2x$ (M1) for setting equation

$-1 = -2x$

$x = \frac{1}{2}$ A1 N2

[2]

(d) 1 A1 N1

[1]

2. (a) $f'(x) = 3x^2 + 2x + 0$ (A1) for correct derivatives

$f'(x) = 3x^2 + 2x$ A1 N2

[2]

(b) $g'(x) = 1$ A1 N1

[1]

(c) $f'(x) = g'(x)$

$3x^2 + 2x = 1$ (M1) for setting equation

$3x^2 + 2x - 1 = 0$

$(x+1)(3x-1) = 0$

$x = -1$ or $x = \frac{1}{3}$ A2 N3

[3]

(d) $x = -\frac{1}{3}$ A1 N1

[1]

3. (a) $f'(x) = 24(1) - 3x^2$ (A1) for correct derivatives

$f'(x) = 24 - 3x^2$ A1 N2

[2]

(b) $g'(x) = 3x^2$ A1 N1

[1]

(c) $f'(x) = g'(x)$

$24 - 3x^2 = 3x^2$ (M1) for setting equation

$24 = 6x^2$

$x^2 = 4$

$x = -2$ or $x = 2$ A2 N3

[3]

(d) The gradient of AB

$= \dfrac{(-2)^3 - 2^3}{-2 - 2}$ (A1) for substitution

$= \dfrac{-16}{-4}$

$= 4$ A1 N2

[2]

4. (a) (i) $f'(x) = -2ax$ A1 N1

(ii) $g'(x) = -4$ A1 N1

[2]

(b) $f'(2) = g'(2)$

$-2a(2) = -4$ (M1) for setting equation

$-4a = -4$

$a = 1$ A1 N2

[2]

(c) 1 A1 N1

[1]

(d) $\dfrac{0-1}{x-2} = -4$ (A1) for substitution

$\dfrac{-1}{x-2} = -4$

$x - 2 = \dfrac{1}{4}$

$x = \dfrac{9}{4}$

Thus, the x-intercept is $\dfrac{9}{4}$. A1 N2

[2]

Exercise 56

1. (a) $f'(x) = 2(3x^2) - 33(2x) + 108(1) - 0$ (A2) for correct derivatives

$f'(x) = 6x^2 - 66x + 108$ A1 N3

[3]

(b) $f'(x) < 0$

$6x^2 - 66x + 108 < 0$ (A1) for correct inequality

$x^2 - 11x + 18 < 0$

By considering the graph of $y = x^2 - 11x + 18$,

$2 < x < 9$. A2 N3

[3]

2. (a) $f'(x) = -4x^3 + 20(3x^2) - 142(2x) + 420(1) + 0$ (A2) for correct derivatives

$f'(x) = -4x^3 + 60x^2 - 284x + 420$ A1 N3

[3]

(b) $f'(x) > 0$

$-4x^3 + 60x^2 - 284x + 420 > 0$ (A1) for correct inequality

By considering the graph of

$y = -4x^3 + 60x^2 - 284x + 420$,

$x < 3$ or $5 < x < 7$. A2 N3

[3]

3. (a) (i) $y = 5$ A1 N1

(ii) $y = 2$ A1 N1

[2]

(b) $x < 3$ or $x > 11$ A2 N2

[2]

(c) (11, 2) A2 N2

[2]

(d) $f(9)$ A1 N1

[1]

4. (a) (i) $y = 0$ A1 N1

(ii) $y = 12$ A1 N1

[2]

(b) $x < -4$, $1 < x < 5$ or $x > 5$ A2 N2

[2]

(c) $(1, 20)$ A2 N2

[2]

(d) $f(2)$ A1 N1

[1]

Exercise 57

1. (a) $C'(x) = 2x + 0 + 54(-x^{-2})$ (A1) for correct derivative

$C'(x) = 2x - \frac{54}{x^2}$ A1 N2

[2]

(b) $C'(x) = 0$

$2x - \frac{54}{x^2} = 0$ (M1) for setting equation

By considering the graph of

$y = 2x - \frac{54}{x^2}$, $x = 3$. (M1) for valid approach

Thus, the required mass is 3 kg. A1 N3

[3]

(c) \$33 A1 N1

[1]

2. (a) 1000 A1 N1

[1]

(b) $P'(t) = 3t^2 - 12(2t) + 36(1) + 0$ (A1) for correct derivative

$P'(t) = 3t^2 - 24t + 36$ A1 N2

[2]

(c) $P'(t) = 0$

$3t^2 - 24t + 36 = 0$ (M1) for setting equation

By considering the graph of

$y = 3t^2 - 24t + 36$, $t = 2$ or $t = 6$ (*Rejected*). (M1) for valid approach

Thus, the required number of days is 2. A1 N3

[3]

(d) 32001000 A1 N1

[1]

3. (a) $Q(t) = 0$

$4t^2 - 120 + \frac{216}{t} = 0$ (M1) for setting equation

By considering the graph of $y = 4t^2 - 120 + \frac{216}{t}$,

$t = 2.1156558$ or $t = 4.1038579$.

Thus, $t = 2.12$ or $t = 4.10$. A2 N3

[3]

(b) $Q'(t) = 4(2t) - 0 + 216(-t^{-2})$ (A1) for correct derivative

$Q'(t) = 8t - \frac{216}{t^2}$ A1 N2

[2]

(c) $Q'(t) = 0$

$8t - \frac{216}{t^2} = 0$ (M1) for setting equation

By considering the graph of $y = 8t - \frac{216}{t^2}$, $t = 3$. (M1) for valid approach

Thus, $t = 3$. A1 N3

[3]

4. (a) $P'(t) = -3t^2 + 9(2t) - 24(1) + 0$ (A1) for correct derivative

$P'(t) = -3t^2 + 18t - 24$ A1 N2

[2]

(b) $P'(t) = 0$

$-3t^2 + 18t - 24 = 0$ (M1) for setting equation

By considering the graph of $y = -3t^2 + 18t - 24$,

$t = 2$ or $t = 4$ (*Rejected*). (M1) for valid approach

Thus, $t = 2$. A1 N3

[3]

(c) The minimum price of the share is \$700, which is greater than \$690. A1 N1

[1]

Exercise 58

1. (a) $r = -(-2)^3 + 3(-2)^2 + 24(-2) - 1$ (A1) for substitution

$r = -29$ A1 N2

[2]

(b) $f'(x) = -3x^2 + 3(2x) + 24(1) - 0$ (A1) for correct derivatives

$f'(x) = -3x^2 + 6x + 24$ A1 N2

[2]

(c) $f'(x) = 0$

$-3x^2 + 6x + 24 = 0$ (M1) for setting equation

$-3(x+2)(x-4) = 0$

$x = -2$ (*Rejected*) or $x = 4$ (A1) for correct value

$f(4) = -4^3 + 3(4)^2 + 24(4) - 1$

$f(4) = 79$

Thus, the required coordinates are $(4, 79)$. A1 N3

[3]

(d) $x < -2$ or $x > 4$ A2 N2

[2]

(e) (i) -1 A1 N1

(ii) $f'(0) = -3(0)^2 + 6(0) + 24$ (M1) for substitution

$f'(0) = 24$ A1 N2

(iii) The equation of tangent:

$y - (-1) = 24(x - 0)$ (M1) for substitution

$y + 1 = 24x$

$24x - y - 1 = 0$ A1 N2

(iv) $24x - 0 - 1 = 0$ (M1) for substitution

$24x = 1$

$x = \frac{1}{24}$ A1 N2

[7]

2. (a) $350 = 2r^3 - 150r - 150$ (M1) for setting equation

$2r^3 - 150r - 500 = 0$

By considering the graph of $y = 2r^3 - 150r - 500$,

$r = -5$. A1 N2

[2]

(b) $f'(x) = 2(3x^2) - 150(1) - 0$ (A1) for correct derivatives

$f'(x) = 6x^2 - 150$ A1 N2

[2]

(c) $f'(x) = 0$

$6x^2 - 150 = 0$ (M1) for setting equation

$6(x+5)(x-5) = 0$

$x = -5$ (*Rejected*) or $x = 5$ (A1) for correct value

$f(5) = 2(5)^3 - 150(5) - 150$

$f(5) = -650$

Thus, the required coordinates are $(5, -650)$. A1 N3

[3]

(d) $x < -5$ or $x > 5$ A2 N2

[2]

(e) (i) -2 A1 N1

(ii) $f'(-1) = 6(-1)^2 - 150$ (M1) for substitution

$f'(-1) = -144$ A1 N2

(iii) $\frac{1}{144}$ A1 N1

(iv) The equation of normal:

$y - (-2) = \frac{1}{144}(x - (-1))$ (M1) for substitution

$y + 2 = \frac{1}{144}x + \frac{1}{144}$

$y = \frac{1}{144}x - \frac{287}{144}$ A1 N2

(v) $0 = \frac{1}{144}x - \frac{287}{144}$ (M1) for substitution

$0 = x - 287$

$x = 287$ A1 N2

[8]

3. (a) $x = 0$ A1 N1

[1]

(b) $f'(x) = 125(1) + 32(-2x^{-3})$ (A1) for correct derivatives

$f'(x) = 125 - \frac{64}{x^3}$ A1 N2

[2]

(c) $f'(x) = 0$

$125 - \frac{64}{x^3} = 0$ (M1) for setting equation

$125 = \frac{64}{x^3}$

$x^3 = \frac{64}{125}$

$x = 0.8$ (A1) for correct value

$f(0.8) = 125(0.8) + \frac{32}{0.8^2}$

$f(0.8) = 150$

Thus, the required coordinates are $(0.8, 150)$. A1 N3

[3]

(d) $0 < x < 0.8$ A2 N2

[2]

(e) (i) 157 A1 N1

(ii) $f'(1) = 125 - \frac{64}{1^3}$ (M1) for substitution

$f'(1) = 61$ A1 N2

(iii) The equation of tangent:

$y - 157 = 61(x - 1)$ (M1) for substitution

$y - 157 = 61x - 61$

$61x - y + 96 = 0$ A1 N2

[5]

(f) $y < 150$ A2 N2

[2]

4. (a) $x = 0$ A1 N1

[1]

(b) $f'(x) = \frac{1}{2}(2x) + 8(-2x^{-3})$ (A1) for correct derivatives

$f'(x) = x - \frac{16}{x^3}$ A1 N2

[2]

(c) $f'(x) = 0$

$x - \frac{16}{x^3} = 0$ (M1) for setting equation

$x = \frac{16}{x^3}$

$x^4 = 16$

$x = 2$ (A1) for correct value

$f(2) = \frac{1}{2}(2)^2 + \frac{8}{2^2}$

$f(2) = 4$

Thus, the required coordinates are $(2, 4)$. A1 N3

[3]

(d) $-1 < x < 0$ or $x > 2$ A2 N2

[2]

(e) (i) 8.5 A1 N1

(ii) $f'(4) = 4 - \frac{16}{4^3}$ (M1) for substitution

$f'(4) = \frac{15}{4}$ A1 N2

(iii) $-\frac{4}{15}$ A1 N1

(iv) The equation of normal:

$y - \frac{17}{2} = -\frac{4}{15}(x - 4)$ (M1) for substitution

$y - \frac{17}{2} = -\frac{4}{15}x + \frac{16}{15}$

$y = -\frac{4}{15}x + \frac{287}{30}$ A1 N2

[6]

(f) $y < 4$ A2 N2

[2]

Exercise 59

1. (a) (i) $\pi r^2 + 2\pi rh = 27\pi$ (M1) for setting equation

$2\pi rh = 27\pi - \pi r^2$

$h = \dfrac{27 - r^2}{2r}$ A1 N2

(ii) $V = \pi r^2 h$

$V = \pi r^2 \left(\dfrac{27 - r^2}{2r} \right)$ A1

$V = \dfrac{\pi r}{2}(27 - r^2)$ A1

$V = \dfrac{27}{2}\pi r - \dfrac{1}{2}\pi r^3$ AG N0

[4]

(b) $\dfrac{\mathrm{d}V}{\mathrm{d}r} = \dfrac{27}{2}\pi(1) - \dfrac{1}{2}\pi(3r^2)$ (A1) for correct derivatives

$\dfrac{\mathrm{d}V}{\mathrm{d}r} = \dfrac{27}{2}\pi - \dfrac{3}{2}\pi r^2$ A1 N2

[2]

(c) $\dfrac{\mathrm{d}V}{\mathrm{d}r} = 0$

$\dfrac{27}{2}\pi - \dfrac{3}{2}\pi r^2 = 0$ (M1) for setting equation

By considering the graph of $y = \dfrac{27}{2}\pi - \dfrac{3}{2}\pi r^2$,

$r = -3$ (*Rejected*) or $r = 3$. (M1) for valid approach

Thus, the required radius is 3 cm. A1 N3

[3]

(d) (i) The maximum volume

$= \dfrac{27}{2}\pi(3) - \dfrac{1}{2}\pi(3)^3$ (A1) for substitution

$= 27\pi \text{ cm}^3$ A1 N2

(ii) $0 \le V \le 27\pi$ A1 N1

[3]

(e) $V = 47$

$\frac{27}{2}\pi r - \frac{1}{2}\pi r^3 = 47$ (M1) for setting equation

$\frac{27}{2}\pi r - \frac{1}{2}\pi r^3 - 47 = 0$

By considering the graph of

$y = \frac{27}{2}\pi r - \frac{1}{2}\pi r^3 - 47$, $r = 1.1670634$ or

$r = 4.5133764$. (M1) for valid approach

Thus, the required radii are 1.17 cm and 4.51 cm. A1 N3

[3]

2. (a) (i) $(64-2x)$ cm A1 N1

(ii) $V = x(64-2x)^2$ A1

$V = x(4096-256x+4x^2)$ A1

$V = 4x^3 - 256x^2 + 4096x$ AG N0

[3]

(b) $\dfrac{dV}{dx} = 4(3x^2) - 256(2x) + 4096(1)$ (A1) for correct derivatives

$\dfrac{dV}{dx} = 12x^2 - 512x + 4096$ A1 N2

[2]

(c) $\dfrac{dV}{dx} = 0$

$12x^2 - 512x + 4096 = 0$ (M1) for setting equation

By considering the graph of

$y = 12x^2 - 512x + 4096$, $x = \dfrac{32}{3}$ or

$x = 32$ (*Rejected*). (M1) for valid approach

Thus, $x = \dfrac{32}{3}$. A1 N3

[3]

(d) (i) The maximum volume

$= 4\left(\dfrac{32}{3}\right)^3 - 256\left(\dfrac{32}{3}\right)^2 + 4096\left(\dfrac{32}{3}\right)$ (A1) for substitution

$= 19418.07407$

$= 19418\,\text{cm}^3$ A1 N2

(ii) $0 \le V \le 19418$ A1 N1

[3]

(e) The total surface area

$= 2\left(64^2 - 4\left(\dfrac{32}{3}\right)^2\right)$ (M1)(A1) for substitution

$= 7281.777778$

$= 7280\,\text{cm}^2$ A1 N3

[3]

3. (a) (i) $100 = 2\left(\frac{1}{2}(r)(r)\sin 60^\circ\right) + 3rh$ A1

$100 = \frac{\sqrt{3}}{2}r^2 + 3rh$ A1

$200 = \sqrt{3}r^2 + 6rh$

$200 - \sqrt{3}r^2 = 6rh$ A1

$h = \frac{200 - \sqrt{3}r^2}{6r}$ AG N0

(ii) $V = \left(\frac{1}{2}(r)(r)\sin 60^\circ\right)h$ A1

$V = \left(\frac{\sqrt{3}}{4}r^2\right)\left(\frac{200 - \sqrt{3}r^2}{6r}\right)$

$V = \left(\frac{\sqrt{3}}{24}r\right)(200 - \sqrt{3}r^2)$ A1

$V = \frac{25\sqrt{3}}{3}r - \frac{1}{8}r^3$ AG N0

[5]

(b) $\frac{\mathrm{d}V}{\mathrm{d}r} = \frac{25\sqrt{3}}{3}(1) - \frac{1}{8}(3r^2)$ (A1) for correct derivatives

$\frac{\mathrm{d}V}{\mathrm{d}r} = \frac{25\sqrt{3}}{3} - \frac{3}{8}r^2$ A1 N2

[2]

(c) $\frac{\mathrm{d}V}{\mathrm{d}r} = 0$

$\frac{25\sqrt{3}}{3} - \frac{3}{8}r^2 = 0$ (M1) for setting equation

By considering the graph of $y = \frac{25\sqrt{3}}{3} - \frac{3}{8}r^2$,

$r = -6.2040324$ (*Rejected*) or $r = 6.2040324$. (M1) for valid approach

Thus, $r = 6.20$. A1 N3

[3]

(d) The maximum volume

$= \frac{25\sqrt{3}}{3}(6.2040324) - \frac{1}{8}(6.2040324)^3$ (A1) for substitution

$= 59.69832955$

$= 59.7 \text{ cm}^3$ A1 N2

[2]

(e) The required density

$$= \frac{9}{59.69832955}$$

$$= 0.1507579872$$

$$= 1.51 \times 10^{-1} \text{ kg/cm}^3$$

(M1) for valid approach

A1 N2

[2]

4. (a) (i) $168 = \left(\frac{\pi r^2}{4}\right)(h)$ A1

$h = \frac{672}{\pi r^2}$ A1 N2

(ii) $A = \left(\frac{\pi r^2}{4}\right)(2) + 2rh + \left(\frac{2\pi r}{4}\right)(h)$ A1

$A = \frac{1}{2}\pi r^2 + 2r\left(\frac{672}{\pi r^2}\right) + \left(\frac{\pi r}{2}\right)\left(\frac{672}{\pi r^2}\right)$ A1

$A = \frac{1}{2}\pi r^2 + \frac{1344}{\pi r} + \frac{336}{r}$ A1

$A = \frac{1}{2}\pi r^2 + \left(\frac{1344}{\pi} + 336\right)\frac{1}{r}$ AG N0

[5]

(b) $\frac{\mathrm{d}A}{\mathrm{d}r} = \frac{1}{2}\pi(2r) + \left(\frac{1344}{\pi} + 336\right)(-r^{-2})$ (A1) for correct derivatives

$\frac{\mathrm{d}A}{\mathrm{d}r} = \pi r - \left(\frac{1344}{\pi} + 336\right)\frac{1}{r^2}$ A1 N2

[2]

(c) $\frac{\mathrm{d}A}{\mathrm{d}r} = 0$

$\pi r - \left(\frac{1344}{\pi} + 336\right)\frac{1}{r^2} = 0$ (M1) for setting equation

By considering the graph of

$y = \pi r - \left(\frac{1344}{\pi} + 336\right)\frac{1}{r^2}$, $r = 6.2413452$. (M1) for valid approach

Thus, $r = 6.24$. A1 N3

[3]

(d) The minimum total surface area

$= \frac{1}{2}\pi(6.2413452)^2 + \left(\frac{1344}{\pi} + 336\right)\frac{1}{6.2413452}$ (A1) for substitution

$= 183.5682368$

$= 184\ \mathrm{cm}^2$ A1 N2

[2]

(e) The minimum number of painting buckets

$= \frac{183.5682368}{25}$ (M1) for valid approach

$= 7.342729471$

Thus, 8 buckets are needed. A1 N2

[2]

Chapter 16 Solution

Exercise 60

1. $f(x) = \int (6x^2 - 2x - 8)\mathrm{d}x$ (M1) for indefinite integral

$f(x) = 6\left(\frac{x^3}{3}\right) - 2\left(\frac{x^2}{2}\right) - 8x + C$ (A2) for correct integration

$f(x) = 2x^3 - x^2 - 8x + C$

$7 = 2(0)^3 - 0^2 - 8(0) + C$ (M1) for substitution

$C = 7$

$\therefore f(x) = 2x^3 - x^2 - 8x + 7$ A1 N5

[5]

2. (a) $f(x) = \int \left(x^2 - 36 - \frac{3}{x^2}\right)\mathrm{d}x$ (M1) for indefinite integral

$f(x) = \frac{1}{3}x^3 - 36x - 3(-x^{-1}) + C$ (A2) for correct integration

$f(x) = \frac{1}{3}x^3 - 36x + \frac{3}{x} + C$

$0 = \frac{1}{3}(3)^3 - 36(3) + \frac{3}{3} + C$ (M1) for substitution

$C = 98$

$\therefore f(x) = \frac{1}{3}x^3 - 36x + \frac{3}{x} + 98$ A1 N5

[5]

(b) $\frac{377}{8} = \frac{1}{3}a^3 - 36a + \frac{3}{a} + 98$ (M1) for setting equation

$\frac{1}{3}a^3 - 36a + \frac{3}{a} + \frac{407}{8} = 0$

By considering the graph of

$y = \frac{1}{3}a^3 - 36a + \frac{3}{a} + \frac{407}{8}$,

$a = -11.03439$ (*Rejected*), $a = 1.5$ or

$a = 9.591087$ (*Rejected*). A1 N2

[2]

3. (a) $f(x)=\int(x^2-64x+12)\mathrm{d}x$ (M1) for indefinite integral

$f(x)=\frac{1}{3}x^3-64\left(\frac{x^2}{2}\right)+12x+C$ (A2) for correct integration

$f(x)=\frac{1}{3}x^3-32x^2+12x+C$

$0=\frac{1}{3}(6)^3-32(6)^2+12(6)+C$ (M1) for substitution

$C=1008$

$\therefore f(x)=\frac{1}{3}x^3-32x^2+12x+1008$ A1 N5

[5]

(b) 1008 A1 N1

[1]

(c) The area of the triangle OAB

$=\frac{(6)(1008)}{2}$ (M1) for valid approach

$=3024$ A1 N2

[2]

4. (a) $f(x)=\int\left(-\frac{200}{x^3}-\frac{20}{x^2}\right)\mathrm{d}x$ (M1) for indefinite integral

$f(x)=-200\left(\frac{x^{-2}}{-2}\right)-20\left(\frac{x^{-1}}{-1}\right)+C$ (A2) for correct integration

$f(x)=\frac{100}{x^2}+\frac{20}{x}+C$

$0=\frac{100}{(-10)^2}+\frac{20}{-10}+C$ (M1) for substitution

$C=1$

$\therefore f(x)=\frac{100}{x^2}+\frac{20}{x}+1$ A1 N5

[5]

(b) $\frac{(0-(-10))(b-0)}{2}=100$ (M1)(A1) for correct formula

$5b=100$

$b=20$ A1 N3

[3]

Exercise 61

1. (a) (i) 10 A1 N1

(ii) (7.67, 50.8) A2 N2

[3]

(b) (i) $\int_3^{10}(-x^3+16x^2-69x+90)\mathrm{d}x$ A1 N1

(ii) $\frac{2401}{12}$ A1 N1

[2]

2. (a) (i) -5 A1 N1

(ii) (5, 0) A2 N2

[3]

(b) (i) $\int_{-5}^{0}(x^3-5x^2-25x+125)\mathrm{d}x$ A1 N1

(ii) $\frac{6875}{12}$ A1 N1

[2]

3. (a) (i) $a=-7,\ b=3$ A2 N2

(ii) $(-2, 50)$ A2 N2

[4]

(b) $\int_{-7}^{c}(-2x^2-8x+42)\mathrm{d}x=\frac{28}{3}$ (M1) for setting equation

$\int_{-7}^{c}(-2x^2-8x+42)\mathrm{d}x-\frac{28}{3}=0$

By considering the graph of

$y=\int_{-7}^{c}(-2x^2-8x+42)\mathrm{d}x-\frac{28}{3},\ c=-6.$ A1 N2

[2]

4. (a) (i) $a = 20,\ b = 40$ A2 N2

(ii) $\int_{20}^{40}(-2x^2 + 120x - 1600)\mathrm{d}x$ A1 N1

(iii) $\frac{8000}{3}$ A1 N1

[4]

(b) $(117 - 17)(c - 30) = \frac{8000}{3}$ (M1)(A1) for correct equation

$c - 30 = \frac{80}{3}$

$c - 30 = \frac{80}{3}$

$c = \frac{170}{3}$ A1 N3

[3]

Exercise 62

1. (a) (i) $\int_{-2}^{10} 1.5(x+2)(x-10)^2 \mathrm{d}x$ A1 N1

(ii) 2592 A1 N1

[2]

(b) The volume of the pyramid

$= \frac{1}{3}(2592)(15)$ (A1) for substitution

$= 12960$ A1 N2

[2]

2. (a) The area of R

$= \int_0^6 \pi(9-(x-3)^2)\mathrm{d}x$ (A1) for correct integral

$= 36\pi$ A1 N2

[2]

(b) $4\pi r^2 = 36\pi$ (M1) for setting equation

$r^2 = 9$

$r = -3$ (*Rejected*) or $r = 3$

Thus, the radius of the sphere is 3. A1 N2

[2]

3. (a) (i) $\int_4^8 (x-4)^2(8-x)\mathrm{d}x$ A1 N1

(ii) $\frac{64}{3}$ A1 N1

[2]

(b) $\frac{64}{3}h = 320$ (M1) for setting equation

$h = 15$ A1 N2

[2]

4. (a) The exact area of R

$= \int_{12}^{16} (x-8)^2 (x-16)^2 \, \mathrm{d}x$ (A1) for correct integral

$= \frac{8192}{15}$ A1 N2

[2]

(b) $\frac{1}{3}\left(\frac{8192}{15} \times 5\right) h = 1024$ (M1) for setting equation

$h = 1.125$ A1 N2

[2]

Exercise 63

1. (a) The width of each trapezium

$= \frac{7-1}{4}$ (A1) for correct substitution

$= 1.5$ A1 N2

[2]

(b) The estimate of $\int_1^7 x^{0.3}\mathrm{d}x$

$= \frac{1}{2}(1.5)\left[1^{0.3} + 7^{0.3} + 2(2.5^{0.3} + 4^{0.3} + 5.5^{0.3})\right]$ (A2) for substitution

$= 8.844238086$

$= 8.84$ A1 N3

[3]

2. (a) $\frac{0.8-0.5}{n} = 0.1$ (A1) for substitution

$\frac{0.3}{n} = 0.1$

$n = 3$ A1 N2

[2]

(b) The estimate of $\int_{0.5}^{0.8} 4^x\mathrm{d}x$

$= \frac{1}{2}(0.1)\left[4^{0.5} + 4^{0.8} + 2(4^{0.6} + 4^{0.7})\right]$ (A2) for substitution

$= 0.7452129098$

$= 0.745$ A1 N3

[3]

3. (a) $\frac{6-a}{6} = 0.4$ (A1) for correct substitution

$6 - a = 2.4$

$a = 3.6$ A1 N2

[2]

(b) The estimate of $\int_a^6 e^x\mathrm{d}x$

$= \frac{1}{2}(0.4)\left[e^{3.6} + e^6 + 2(e^4 + e^{4.4} + e^{4.8} + e^{5.2} + e^{5.6})\right]$ (A2) for substitution

$= 371.7086398$

$= 372$ A1 N3

[3]

4. (a) $\int_2^b \frac{1}{2x}\,dx$ A1 N1

[1]

(b) $\frac{b-2}{8} = 0.25$ (A1) for correct substitution

$b-2=2$

$b=4$ A1 N2

[2]

(c) The estimate of $\int_2^b \frac{1}{2x}\,dx$

$$= \frac{1}{2}(0.25)\left[\begin{array}{l}\frac{1}{2(2)}+\frac{1}{2(4)} \\ +2\left(\begin{array}{l}\frac{1}{2(2.25)}+\frac{1}{2(2.5)}+\frac{1}{2(2.75)} \\ +\frac{1}{2(3)}+\frac{1}{2(3.25)}+\frac{1}{2(3.5)} \\ +\frac{1}{2(3.75)}\end{array}\right)\end{array}\right]$$ (A2) for substitution

$=0.3470609252$

$=0.347$ A1 N3

[3]

Exercise 64

1. (a) The estimate of $\int_{-11}^{-8} \frac{3}{\sqrt{x+12}}\,dx$

$$= \frac{1}{2}(0.75)\left[\begin{array}{l}\frac{3}{\sqrt{-11+12}}+\frac{3}{\sqrt{-8+12}} \\ +2\left(\frac{3}{\sqrt{-10.25+12}}+\frac{3}{\sqrt{-9.5+12}}\right. \\ \left.+\frac{3}{\sqrt{-8.75+12}}\right)\end{array}\right]$$ (A2) for substitution

$= 6.059440517$

$= 6.06$ A1 N3

[3]

(b) The percentage error

$= \left|\frac{6.059440517-6}{6}\right| \times 100\%$ (A1) for correct substitution

$= 0.9906752854\%$

$= 0.991\%$ A1 N2

[2]

2. (a) 0.6 A1 N1

[1]

(b) The estimate of $\int_0^3 4e^x dx$

$= \frac{1}{2}(0.6)\left[4e^0+4e^3+2(4e^{0.6}+4e^{1.2}+4e^{1.8}+4e^{2.4})\right]$ (A2) for substitution

$= 78.61878727$

$= 78.6$ A1 N3

[3]

(c) The percentage error

$= \left|\frac{78.61878727-4(e^3-1)}{4(e^3-1)}\right| \times 100\%$ (A1) for correct substitution

$= 2.982152908\%$

$= 2.98\%$ A1 N2

[2]

3. (a) (i) $\frac{5}{3}$ A1 N1

(ii) $f(5.75)=\frac{14}{3},\ f(5.875)=\frac{19}{3}$ A2 N2

[3]

(b) The estimate of $\int_5^6 f(x)\mathrm{d}x$

$=\frac{1}{2}(0.125)\left[8+8+2\left(7+6+5+4+3+\frac{14}{3}+\frac{19}{3}\right)\right]$ (A2) for substitution

$=5.5$ A1 N3

[3]

(c) Overestimate A1 N1

[1]

4. (a) $a=0.35,\ b=0.5$ A2 N2

[2]

(b) The estimate of $\int_{0.3}^{0.6} f(x)\mathrm{d}x$

$=\frac{1}{2}(0.05)\begin{bmatrix}0.25^{0.3}+0.25^{0.6}\\+2(0.25^{0.35}+0.25^{0.4}\\+0.25^{0.45}+0.5+0.25^{0.55})\end{bmatrix}$ (A2) for substitution

$=0.1619919615$

$=0.162$ A1 N3

[3]

(c) Overestimate A1 N1

[1]

Chapter 17 Solution

Exercise 65

1. (a) The mean

$$= \frac{8+9+14+18+2+5+1+0+0+10+6+11}{12}$$ (A1) for correct formula

$= 7$ A1 N2

[2]

(b) 7 A1 N1

[1]

(c) The inter-quartile range

$= 10.5 - 1.5$ (A1) for correct formula

$= 9$ A1 N2

[2]

(d) 5.51 A1 N1

[1]

2. (a) The mean

$$= \frac{3+8+6+10+2+7+7+9}{8}$$ (A1) for correct formula

$= 6.5$ A1 N2

[2]

(b) (i) 7 A1 N1

(ii) 8.5 A1 N1

(iii) 4.5 A1 N1

(iv) 4 A1 N1

[4]

3. (a) $\frac{7+6+12+3+6+8+11+5+4+x}{10}=7$ (A1) for correct formula

$62+x=70$

$x=8$ A1 N2

[2]

(b) 6, 8 A2 N2

[2]

(c) The inter-quartile range

$=8-5$ (A1) for correct formula

$=3$ A1 N2

[2]

4. (a) (i) 48 A1 N1

(ii) 59 A1 N1

(iii) $\frac{(50+x)+62}{2}=59$ (A1) for correct formula

$112+x=118$

$x=6$ A1 N2

[4]

(b) 19 A1 N1

[1]

(c) 6.29 A1 N1

[1]

Exercise 66

1. (a) (i) 27.5 — A1 N1

(ii) $30 \le x < 35$ — A1 N1

[2]

(b) (i) 34.625 — A2 N2

(ii) 6.06 — A1 N1

(iii) 36.7 — A1 N1

[4]

2. (a) $87+55+27+11+f+5=200$ — (M1) for setting equation

$f=15$ — A1 N2

[2]

(b) (i) Continuous — A1 N1

(ii) 175 USD — A1 N1

(ii) $0 \le x < 50$ — A1 N1

[3]

(c) (i) 81.75 USD — A2 N2

(ii) 67.9 USD — A1 N1

[3]

3. (a) 12 — A1 N1

[1]

(b) (i) 18 — A1 N1

(ii) $12 \le x < 16$ — A1 N1

[2]

(c) (i) 11.1 — A2 N2

(ii) 4.95 — A1 N1

[3]

(d) The upper bound for the greatest possible score

$=11.11111111+4.954110402$ — (A1) for correct approach

$=16.06522151$

Thus, the greatest possible score is 16. — A1 N2

[2]

4. (a) $p = 6,\ q = 5,\ r = 3,\ s = 2$

For any two correct answers A1

For all correct answers A1 N2

[2]

(b) $0 \le x < 3$ A1 N1

[1]

(c) (i) 2.87 A1 N1

(ii) 3.09 A2 N2

(iii) The percentage error

$= \left| \dfrac{3.086639556 - 2.871601252}{2.871601252} \right| \times 100\%$ (A1) for correct substitution

$= 7.488445823\%$

$= 7.49\%$ A1 N2

[5]

Exercise 67

1. (a) (i) $p = 14 + 7$

$p = 21$ A1 N1

(ii) $q = 39 - 21$ (M1) for valid approach

$q = 18$ A1 N2

[3]

(b) The mean number of notebooks

$= \dfrac{(1)(14) + (2)(7) + (3)(18) + (4)(10) + (5)(1)}{50}$ (A1) for correct formula

$= 2.54$ A1 N2

[2]

(c) 1.15 A1 N1

[1]

2. (a) (i) $p = 53 + 37$

$p = 90$ A1 N1

(ii) $q = 165 - 115$ (M1) for valid approach

$q = 50$ A1 N2

[3]

(b) The mean number of sit-ups

$= \dfrac{(22)(32) + (23)(21) + (24)(37) + (25)(25) + (26)(50) + (27)(15)}{180}$ (A1) for correct formula

$= 24.47222222$

$= 24.5$ A1 N2

[2]

(c) 2.60 A1 N1

[1]

3. (a) $\dfrac{(1)(2)+(2)(4)+(3)(6)+(4)(16)+5p+(6)(10)}{2+4+6+16+p+10}=4.24$ (M1)(A1) for correct formula

$\dfrac{5p+152}{p+38}=4.24$ (A1) for simplification

$5p+152=4.24p+161.12$

$0.76p=9.12$

$p=12$ A1 N4

[4]

(b) $q=12+28+10$ (M1) for valid approach

$q=50$ A1 N2

[2]

(c) Discrete A1 N1

[1]

4. (a) $\dfrac{(7)(5)+(12)(3)+(17)(6)+(22)(5)+27p}{5+3+6+5+p}=17.8$ (M1)(A1) for correct formula

$\dfrac{27p+283}{p+19}=17.8$ (A1) for simplification

$27p+283=17.8p+338.2$

$9.2p=55.2$

$p=6$ A1 N4

[4]

(b) The upper quartile

$=\dfrac{19\text{th}+20\text{th}}{2}$ (M1) for valid approach

$=\dfrac{22+27}{2}$

$=24.5$ A1 N2

[2]

Exercise 68

1. (a) $a = 3$ A1 N1
$b = 14$ A1 N1
[2]

(b) $p > 14 + 1.5(6)$ (M1)(A1) for correct inequality
$p > 23$ (A1) for correct value
Thus, the least value of p is 24. A1 N4
[4]

2. (a) $a = 63$ A1 N1
$b = 73$ A1 N1
[2]

(b) $k > 73 + 1.5(10)$ (M1)(A1) for correct inequality
$k > 88$ (A1) for correct value
Thus, the least value of k is 89. A1 N4
[4]

3. (a) (i) 34 A1 N1

(ii) 24 A1 N1

(iii) 12 A1 N1
[3]

(b) As the median is 34, the number of data less than 34 is the same as that of greater than 34. (R1) for valid argument
$\therefore 2 + 4 = q + 1$ (A1) for correct equation
$q = 5$ A1 N3
[3]

4. (a) (i) 5 A1 N1

(ii) 8 A1 N1

(iii) 6 A1 N1
[3]

(b) As the median is 5, the number of data less than 5 is the same as that of greater than 5. (R1) for valid argument
$\therefore 1 + r = 5 + 3 + 2$ (A1) for correct equation
$r = 9$ A1 N3
[3]

Exercise 69

1. (a) The mean

$= \frac{150}{15}$ (A1) for correct formula

$= 10$ A1 N2

[2]

(b) (i) 30 A1 N1

(ii) The new variance

$= (3^2)(8)$ (M1) for valid approach

$= 72$ A1 N2

[3]

2. (a) The sum of the items

$= (12)(9)$ (A1) for correct formula

$= 108$ A1 N2

[2]

(b) (i) 19 A1 N1

(ii) The new standard deviation

$= \sqrt{2.25}$ (M1) for valid approach

$= 1.5$ A1 N2

[3]

3. (a) The upper quartile

$= \frac{20 + 22}{2}$ (A1) for correct formula

$= 21$ A1 N2

[2]

(b) (i) 40 A1 N1

(ii) The new inter-quartile range

$= 4(21 - 10)$ (M1) for valid approach

$= 44$ A1 N2

[3]

4. (a) The lower quartile

$= \frac{8+12}{2}$ (A1) for correct formula

$= 10$ A1 N2

[2]

(b) (i) 19 A1 N1

(ii) The new upper quartile

$= 10 + 5 + 19$ (M1) for valid approach

$= 34$ A1 N2

[3]

Exercise 70

1. (a) (i) \$7.5 A2 N2

(ii) 20 A1 N1

[3]

(b) (i) The number of learning points

$= (5)(15)$

$= 75$ A1 N1

(ii) The number of learning points

$= (5)(15) + (10)(20 - 15)$ (M1)(A1) for correct formula

$= 125$ A1 N3

[4]

(c) The amount raised

$= \frac{62.5}{5}$ (A1) for correct formula

$= \$12.5$

Thus, the number of students

$= 120 - 50$ (A1) for correct formula

$= 70$ A1 N3

[3]

(d) The number of students awarded not more than k learning points

$= 120 - 80$ (M1) for valid approach

$= 40$ (A1) for correct value

$k = (5)(10)$ (A1) for correct formula

$k = 50$ A1 N4

[4]

(e) Simple random sampling A1 N1

[1]

2. (a) (i) 1.5 cm A2 N2

(ii) 20 A1 N1

(iii) The percentage of fish

$= \frac{100-20}{200} \times 100\%$ (M1) for valid approach

$= 40\%$ A1 N2

(iv) The number of fish not longer than k cm

$= 200 \times (1-90\%)$ (M1) for valid approach

$= 20$ (A1) for correct value

$\therefore k = 1$ A1 N3

[8]

(b) The price

$= (20)(4.5)$ (A1) for correct formula

$= \$90$ A1 N2

[2]

(c) The number of fish

$= 200 \times (1-10\%)$ (A1) for correct formula

$= 180$

180 fish are not longer than 4 cm.

Thus, 20 fish are longer than 4 cm. (A1) for correct value

$r = (20)(4)$ (A1) for correct formula

$r = 80$ A1 N4

[4]

3. (a) 25 minutes — A2 N2

[2]

(b) 15 minutes — A2 N2

[2]

(c) The number of students whose travelling time is within 5 minutes of the median
$=$ The number of students whose travelling time is between 20 minutes and 30 minutes — (M1) for valid approach
$=120-60$ — (A1) for correct formula
$=60$ — A1 N3

[3]

(d) The number of students spent not more than k minutes to travel to school
$=160-160\times 6.25\%$ — (A1) for correct formula
$=160-10$
$=150$ — (A1) for correct value
$\therefore k=40$ — A1 N3

[3]

(e) $r=30+(1.5)(15)$ — (M1)(A1) for correct formula
$r=52.5$ — A1 N3

[3]

(f) Systematic sampling — A1 N1

[1]

4. (a) 35 minutes A2 N2

[2]

(b) 10 minutes A2 N2

[2]

(c) The number of secretaries whose time for presentation is within 5 minutes of the upper quartile
$=$ The number of secretaries whose time for presentation is between 35 minutes and 45 minutes (M1) for valid approach
$= 70 - 40$ (A1) for correct formula
$= 30$ A1 N3

[3]

(d) The number of secretaries spent not more than k minutes to complete a presentation
$= 80(1 - 87.5\%)$ (A1) for correct formula
$= 100$ (A1) for correct value
$\therefore k = 25$ A1 N3

[3]

(e) $r = 40 + (1.5)(10)$ (M1)(A1) for correct formula
$r = 55$ A1 N3

[3]

(f) The probability
$= \dfrac{80 - 75}{80}$ (M1) for valid approach
$= \dfrac{1}{16}$ A1 N2

[2]

Chapter 18 Solution

Exercise 71

1. (a) (i) The required probability

$$= \frac{3+2+4+3}{20}$$ (A1) for correct formula

$$= \frac{3}{5}$$ A1 N2

(ii) The required probability

$$= \frac{3+5}{3+3+5}$$ (A1) for correct formula

$$= \frac{8}{11}$$ A1 N2

[4]

(b) The required probability

$$= \left(\frac{3+2+3+3}{20}\right)\left(\frac{3+2+3+3-1}{20-1}\right)$$ (A2) for correct formula

$$= \left(\frac{11}{20}\right)\left(\frac{10}{19}\right)$$

$$= \frac{11}{38}$$ A1 N3

[3]

2. (a) (i) The required probability

$$= \frac{2+10+3+5+10}{50}$$ (A1) for correct formula

$$= \frac{3}{5}$$ A1 N2

(ii) The required probability

$$= \frac{3+5+10}{10+3+5+10}$$ (A1) for correct formula

$$= \frac{9}{14}$$ A1 N2

[4]

(b) The required probability

$$= \left(\frac{5+10}{50}\right)\left(\frac{5+10-1}{50-1}\right)$$ (A2) for correct formula

$$= \left(\frac{15}{50}\right)\left(\frac{14}{49}\right)$$

$$= \frac{3}{35}$$ A1 N3

[3]

3. (a) (i) The required probability

$$= \frac{2+1+5+3+4+2+1}{25}$$ (A1) for correct formula

$$= \frac{18}{25}$$ A1 N2

(ii) The required probability

$$= \frac{5}{1+5+2}$$ (A1) for correct formula

$$= \frac{5}{8}$$ A1 N2

[4]

(b) The required probability

$$= \left(\frac{4+2+1}{25}\right)\left(\frac{4+2+1-1}{25-1}\right)$$ (A2) for correct formula

$$= \left(\frac{7}{25}\right)\left(\frac{6}{24}\right)$$

$$= \frac{7}{100}$$ A1 N3

[3]

4. (a) $\dfrac{5+15+a}{100}=\dfrac{6}{25}$ (M1) for setting equation

$20+a=24$

$a=4$ A1

$5+15+4+\cdots+15+b=100$

$b=6$ A1 N3

[3]

(b) The required probability

$=\dfrac{15+4+10+10+15+6}{15+4+5+5+10+10+15+6}$ (A1) for correct formula

$=\dfrac{6}{7}$ A1 N2

[2]

(c) The required probability

$=\left(\dfrac{6}{100}\right)\left(\dfrac{6-1}{100-1}\right)$ (A2) for correct formula

$=\left(\dfrac{6}{100}\right)\left(\dfrac{5}{99}\right)$

$=\dfrac{1}{330}$ A1 N3

[3]

Exercise 72

1. (a) (i) $a+9=13$ — (M1) for valid approach
$a=4$ — A1 N2

(ii) $21+4+b=30$ — (M1) for valid approach
$b=5$ — A1 N2

[4]

(b) The required probability
$=\frac{4}{30}$ — (M1) for valid approach
$=\frac{2}{15}$ — A1 N2

[2]

2. (a) (i) $17+15-h+10=40$ — (M1) for valid approach
$h=2$ — A1 N2

(ii) $2+k=15$ — (M1) for valid approach
$k=13$ — A1 N2

[4]

(b) The required probability
$=\frac{2}{40}$ — (M1) for valid approach
$=\frac{1}{20}$ — A1 N2

[2]

3. (a) (i) $p=0.4$ — A1 N1

(ii) $0.4+q=0.6$ — (M1) for valid approach
$q=0.2$ — A1 N2

[3]

(b) $\mathrm{P}(A\cup B)=\mathrm{P}(A)+\mathrm{P}(B)-\mathrm{P}(A\cap B)$ — (M1) for valid approach
$0.9=\mathrm{P}(A)+0.6-0.4$ — (A1) for substitution
$\mathrm{P}(A)=0.7$ — A1 N3

[3]

4. (a) (i) $a = 0.3$ A1 N1

(ii) $0.3 + b = 1 - 0.6$ (M1) for valid approach
$b = 0.1$ A1 N2

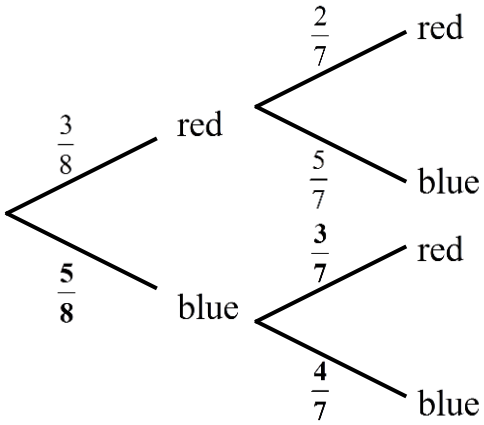
[3]

(b) $\mathrm{P}(A \cup B) = \mathrm{P}(A) + \mathrm{P}(B) - \mathrm{P}(A \cap B)$ (M1) for valid approach
$1 - 0.3 = 0.6 + 0.2 - \mathrm{P}(A \cap B)$ (A1) for substitution
$\mathrm{P}(A \cap B) = 0.1$ A1 N3

[3]

Exercise 73

1. (a)



A3 N3

[3]

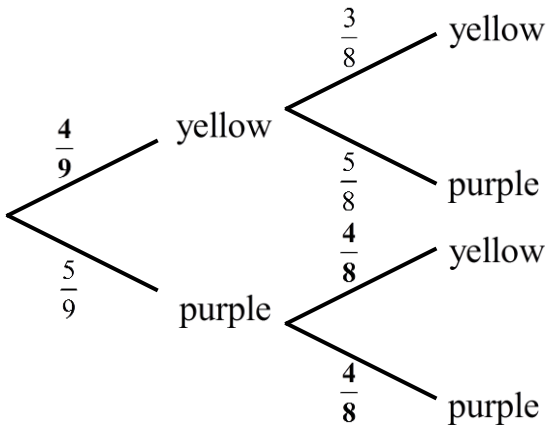
(b) The required probability

$$=\left(\frac{3}{8}\right)\left(\frac{5}{7}\right)+\left(\frac{5}{8}\right)\left(\frac{3}{7}\right)$$ (M1)(A1) for correct formula

$$=\frac{15}{28}$$ A1 N3

[3]

2. (a)



A3 N3

[3]

(b) The required probability

$=\left(\frac{4}{9}\right)\left(\frac{5}{8}\right)+\frac{5}{9}$ (M1)(A1) for correct formula

$=\frac{5}{6}$ A1 N3

[3]

3. (a) $x=\frac{5}{8}$ A1 N1

[1]

(b) $\mathrm{P}(B)=\left(\frac{3}{8}\right)\left(\frac{1}{5}\right)+\left(\frac{5}{8}\right)\left(\frac{2}{5}\right)$ (M1)(A1) for correct formula

$\mathrm{P}(B)=\frac{13}{40}$ A1 N3

[3]

(c) $\mathrm{P}(A\,|\,B)=\frac{\mathrm{P}(A\cap B)}{\mathrm{P}(B)}$ (M1) for valid approach

$\mathrm{P}(A\,|\,B)=\frac{\left(\frac{3}{8}\right)\left(\frac{1}{5}\right)}{\frac{13}{40}}$ (A1) for substitution

$\mathrm{P}(A\,|\,B)=\frac{3}{13}$ A1 N3

[3]

4. (a) $\mathrm{P}(B \mid A') = \frac{3}{5}$ A1 N1

[1]

(b) $\mathrm{P}(B) = \left(\frac{1}{3}\right)\left(\frac{4}{5}\right) + \left(\frac{2}{3}\right)\left(\frac{3}{5}\right)$ (M1)(A1) for correct formula

$\mathrm{P}(B) = \frac{2}{3}$ A1 N3

[3]

(c) $\mathrm{P}(A' \mid B) = \frac{\mathrm{P}(A' \cap B)}{\mathrm{P}(B)}$ (M1) for valid approach

$\mathrm{P}(A' \mid B) = \frac{\left(\frac{2}{3}\right)\left(\frac{3}{5}\right)}{\frac{2}{3}}$ (A1) for substitution

$\mathrm{P}(A' \mid B) = \frac{3}{5}$ A1 N3

[3]

Exercise 74

1. (a) $P(A) = P(A \cap B) + P(A \cap B')$ (M1) for valid approach

$P(A) = 0.08 + 0.12$

$P(A) = 0.2$ A1 N2

[2]

(b) $P(A \cap B) = P(A) \times P(B)$

$0.08 = 0.2 \times P(B)$ (A1) for substitution

$P(B) = 0.4$ A1 N2

[2]

(c) $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$P(A \cup B) = 0.2 + 0.4 - 0.08$ (A1) for substitution

$P(A \cup B) = 0.52$ A1 N2

[2]

2. (a) $P(B) = P(A \cap B) + P(A' \cap B)$ (M1) for valid approach

$0.3 = P(A \cap B) + 0.15$

$P(A \cap B) = 0.15$ A1 N2

[2]

(b) $P(A \cap B) = P(A) \times P(B)$

$0.15 = P(A) \times 0.3$ (A1) for substitution

$P(A) = 0.5$ A1 N2

[2]

(c) $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$P(A \cup B) = 0.5 + 0.3 - 0.15$ (A1) for substitution

$P(A \cup B) = 0.65$ A1 N2

[2]

3. (a) $P(A) = P(A \cap B) + P(A \cap B')$ (M1) for valid approach

$0.4 = P(A \cap B) + 0.28$

$P(A \cap B) = 0.12$ A1 N2

[2]

(b) $P(A \cap B) = P(A) \times P(B)$

$0.12 = 0.4 \times P(B)$ (A1) for substitution

$P(B) = 0.3$ (A1) for correct value

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$P(A \cup B) = 0.4 + 0.3 - 0.12$ (A1) for substitution

$P(A \cup B) = 0.58$ A1 N4

[4]

4. (a) $P(A \cap B) = P(A) \times P(B)$ (M1) for valid approach

$0.21 = 0.7P(B)$

$P(B) = 0.3$ A1 N2

[2]

(b) $P(A \cup B) = P(A) + P(B) - P(A \cap B)$ M1

$P(A \cup B) = 0.7 + 0.3 - 0.21$ A1

$P(A \cup B) = 0.79$ AG N0

[2]

(c) $P(A' \cap B') = 1 - P(A \cup B)$ (M1) for valid approach

$P(A' \cap B') = 1 - 0.79$ (A1) for substitution

$P(A' \cap B') = 0.21$ A1 N3

[3]

Exercise 75

1. (a) $\mathrm{P}(C \cap D) = \mathrm{P}(C) \times \mathrm{P}(D)$

$\mathrm{P}(C \cap D) = 2k^2 \times 3k^2$ (A1) for substitution

$\mathrm{P}(C \cap D) = 6k^4$ A1 N2

[2]

(b) $6k^4 = 0.0096$ (A1) for correct equation

$k^4 = 0.0016$

$k = 0.2$ A1 N2

[2]

(c) $\mathrm{P}(C) = \mathrm{P}(C \cap D) + \mathrm{P}(C \cap D')$

$2(0.2)^2 = 6(0.2)^4 + \mathrm{P}(C \cap D')$ (A1) for substitution

$\mathrm{P}(C \cap D') = 0.0704$ A1 N2

[2]

(d) $\mathrm{P}(D' \mid C) = \dfrac{\mathrm{P}(D' \cap C)}{\mathrm{P}(C)}$

$\mathrm{P}(D' \mid C) = \dfrac{0.0704}{2(0.2)^2}$ (A1) for substitution

$\mathrm{P}(D' \mid C) = 0.88$ A1 N2

[2]

2. (a) $\mathrm{P}(E \cap F) = \mathrm{P}(E) \times \mathrm{P}(F)$

$\mathrm{P}(E \cap F) = 4k^3 \times k$ (A1) for substitution

$\mathrm{P}(E \cap F) = 4k^4$ A1 N2

[2]

(b) $4k^4 = \dfrac{1}{2500}$ (A1) for correct equation

$k^4 = \dfrac{1}{10000}$

$k = \dfrac{1}{10}$ A1 N2

[2]

(c) $\mathrm{P}(E \cup F) = \mathrm{P}(E) + \mathrm{P}(F) - \mathrm{P}(E \cap F)$

$\mathrm{P}(E \cup F) = 4\left(\dfrac{1}{10}\right)^3 + \dfrac{1}{10} - 4\left(\dfrac{1}{10}\right)^4$ (A1) for substitution

$\mathrm{P}(E \cup F) = \dfrac{259}{2500}$ A1 N2

[2]

3. (a) $P(A \cap B) = P(A) \times P(B)$ M1

$P(A \cap B) = 2k \times 1.5(2k)$ A1

$P(A \cap B) = 6k^2$ AG N0

[2]

(b) $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$6k - 1 = 2k + 1.5(2k) - 6k^2$ A1

$6k - 1 = 5k - 6k^2$ A1

$6k^2 + k - 1 = 0$ AG N0

[2]

(c) $6k^2 + k - 1 = 0$

$(3k - 1)(2k + 1) = 0$ (A1) for factorization

$k = \frac{1}{3}$ or $k = -\frac{1}{2}$ (*Rejected*) A1 N2

[2]

(d) $P(B \mid A) = \frac{P(A \cap B)}{P(A)}$

$P(B \mid A) = \frac{6\left(\frac{1}{3}\right)^2}{2\left(\frac{1}{3}\right)}$ (A1) for substitution

$P(B \mid A) = 1$ A1 N2

[2]

4. (a) $P(A \cap B) = P(A) \times P(B)$

$P(A \cap B) = (x)(3x)$ A1

$P(A \cap B) = 3x^2$ A1

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$ M1

$0.93 = x + 3x - 3x^2$ A1

$3x^2 - 4x + 0.93 = 0$

$300x^2 - 400x + 93 = 0$ AG N0

[4]

(b) $300x^2 - 400x + 93 = 0$

$(30x - 31)(10x - 3) = 0$ (A1) for factorization

$x = \frac{31}{30}$ (*Rejected*) or $x = \frac{3}{10}$ (A1) for correct value

$P(B) = 3\left(\frac{3}{10}\right) = \frac{9}{10}$ A1 N3

[3]

Exercise 76

1. (a) $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$1 = 0.4 + 0.65 - P(A \cap B)$ (A1) for substitution

$P(A \cap B) = 0.05$ A1 N2

[2]

(b) $P(A \cap B) + P(A' \cap B) = P(B)$ (M1) for valid approach

$0.05 + P(A' \cap B) = 0.65$

$P(A' \cap B) = 0.6$ A1 N2

[2]

(c) (i) $P(A \cap C) = P(A \mid C) \times P(C)$ (M1) for valid approach

$P(A \cap C) = 0.78 \times 0.7$ (A1) for substitution

$P(A \cap C) = 0.546$ A1 N3

(ii) $P(A \cap C) + P(A' \cap C) = P(C)$ (M1) for valid approach

$0.546 + P(A' \cap C) = 0.7$

$P(A' \cap C) = 0.154$ A1 N2

(iii) $P(A) + P(A') = 1$

$0.4 + P(A') = 1$ (A1) for substitution

$P(A') = 0.6$ A1 N2

(iv) $P(C \mid A') = \dfrac{P(C \cap A')}{P(A')}$

$P(C \mid A') = \dfrac{0.154}{0.6}$ (A1) for substitution

$P(C \mid A') = 0.256666666$

$P(C \mid A') = 0.257$ A1 N2

[9]

2. (a) $\mathrm{P}(A \cup T) = \mathrm{P}(A) + \mathrm{P}(T) - \mathrm{P}(A \cap T)$

$1 = 0.55 + 0.7 - \mathrm{P}(A \cap T)$ (A1) for substitution

$\mathrm{P}(A \cap T) = 0.25$

Thus, the required percentage is 25%. A1 N2

[2]

(b) $\mathrm{P}(A \cup T) - \mathrm{P}(A \cap T)$ (M1) for valid approach

$= 1 - 0.25$

$= 0.75$

Thus, the required percentage is 75%. A1 N2

[2]

(c) (i) The required probability

$= \mathrm{P}(M \cap A)$

$= \mathrm{P}(A \mid M) \times \mathrm{P}(M)$ (M1) for valid approach

$= 0.72 \times 0.63$ (A1) for substitution

$= 0.4536$ A1 N3

(ii) $\mathrm{P}(M \cap A) + \mathrm{P}(M' \cap A) = \mathrm{P}(A)$ (M1) for valid approach

$0.4536 + \mathrm{P}(M' \cap A) = 0.55$

$\mathrm{P}(M' \cap A) = 0.0964$

Thus, the required probability is 0.0964. A1 N2

(iii) $\mathrm{P}(M) + \mathrm{P}(M') = 1$

$0.63 + \mathrm{P}(M') = 1$ (A1) for substitution

$\mathrm{P}(M') = 0.37$

Thus, the required probability is 0.37. A1 N2

(iv) The required probability

$= \mathrm{P}(A \mid M')$

$= \dfrac{\mathrm{P}(A \cap M')}{\mathrm{P}(M')}$

$= \dfrac{0.0964}{0.37}$ (A1) for substitution

$= 0.26054054$

$= 0.261$ A1 N2

[9]

3. (a) $\mathrm{P}(F \cup R) = \mathrm{P}(F) + \mathrm{P}(R) - \mathrm{P}(F \cap R)$

$1 = 0.85 + 0.45 - \mathrm{P}(F \cap R)$ (A1) for substitution

$\mathrm{P}(F \cap R) = 0.3$

Thus, the required percentage is 30%. A1 N2

[2]

(b) $\mathrm{P}(F \cap R) + \mathrm{P}(F' \cap R) = \mathrm{P}(R)$ (M1) for valid approach

$0.3 + \mathrm{P}(F' \cap R) = 0.45$

$\mathrm{P}(F' \cap R) = 0.15$

Thus, the required percentage is 15%. A1 N2

[2]

(c) $\mathrm{P}(F \cup R) - \mathrm{P}(F \cap R) = 1 - 0.3$ (M1) for valid approach

$\mathrm{P}(F \cup R) - \mathrm{P}(F \cap R) = 0.7$

Thus, the required percentage is 70%. A1 N2

[2]

(d) (i) The required probability

$= \mathrm{P}(R \mid F)$

$= \dfrac{\mathrm{P}(R \cap F)}{\mathrm{P}(F)}$

$= \dfrac{0.3}{0.85}$ (M1) for substitution

$= 0.352941176$

$= 0.353$ A1 N2

(ii) The required probability

$= \dfrac{0.15}{0.7}$ (M1) for substitution

$= 0.214285714$

$= 0.214$ A1 N2

[4]

(e) (i) 54% A1 N1

(ii) $\mathrm{P}(F \cap T) + \mathrm{P}(F \cap T') = \mathrm{P}(F)$ (M1) for valid approach

$0.54 + \mathrm{P}(F \cap T') = 0.85$

$\mathrm{P}(F \cap T') = 0.31$

Thus, the required percentage is 31%. A1 N2

[3]

4. (a) $\mathrm{P}(F \cup T) = \mathrm{P}(F) + \mathrm{P}(T) - \mathrm{P}(F \cap T)$

$1 - 0.25 = 0.35 + 0.5 - \mathrm{P}(F \cap T)$ (A1) for substitution

$\mathrm{P}(F \cap T) = 0.1$

Thus, the required percentage is 10%. A1 N2

[2]

(b) $\mathrm{P}(F \cap T) + \mathrm{P}(F' \cap T) = \mathrm{P}(T)$ (M1) for valid approach

$0.1 + \mathrm{P}(F' \cap T) = 0.5$

$\mathrm{P}(F' \cap T) = 0.4$

Thus, the required percentage is 40%. A1 N2

[2]

(c) 75% A1 N1

[1]

(d) (i) The required probability

$= \dfrac{0.4}{0.75}$ (A1) for substitution

$= 0.533333333$

$= 0.533$ A1 N2

(ii) The required probability

$= \mathrm{P}(Q \mid T)$

$= \dfrac{\mathrm{P}(Q \cap T)}{\mathrm{P}(T)}$

$= \dfrac{0.1}{0.5}$ (A1) for substitution

$= 0.2$ A1 N2

[4]

(e) (i) 38% A1 N1

(ii) $\mathrm{P}(T \cap G) + \mathrm{P}(T \cap G') = \mathrm{P}(T)$ (M1) for valid approach

$0.38 + \mathrm{P}(T \cap G') = 0.5$

$\mathrm{P}(T \cap G') = 0.12$

Thus, the required percentage is 12%. A1 N2

[3]

Chapter 19 Solution

Exercise 77

1. (a) $P(X=0)+P(X=1)+P(X=2)+P(X=3)=1$ (A1) for correct formula

$9k+k+0.1+0.4=1$

$10k=0.5$

$k=0.05$ A1 N2

[2]

(b) $E(X)=9k(0)+k+0.1(2)+0.4(3)$ (A1) for correct formula

$E(X)=0+0.05+0.2+1.2$ (A1) for substitution

$E(X)=1.45$ A1 N3

[3]

2. (a) $P(X=0)+P(X=20)+P(X=40)+P(X=60)=1$ (A1) for correct formula

$\frac{1}{10}+\frac{1}{5}+\frac{2}{5}+k=1$

$\frac{7}{10}+k=1$

$k=\frac{3}{10}$ A1 N2

[2]

(b) $E(X)=\frac{1}{10}(0)+\frac{1}{5}(20)+\frac{2}{5}(40)+60k$ (A1) for correct formula

$E(X)=0+4+16+60\left(\frac{3}{10}\right)$ (A1) for substitution

$E(X)=38$ A1 N3

[3]

3. (a) $\mathrm{P}(X=1)+\mathrm{P}(X=2)+\mathrm{P}(X=k)=1$ (A1) for correct formula

$\frac{1}{14}+\frac{4}{14}+\frac{k^2}{14}=1$

$5+k^2=14$ (A1) for simplification

$k^2=9$

$k=3$ A1 N3

[3]

(b) $\mathrm{E}(X)=\frac{1}{14}(1)+\frac{4}{14}(2)+\frac{k^2}{14}(k)$ (A1) for correct formula

$\mathrm{E}(X)=\frac{1}{14}+\frac{8}{14}+\frac{3^3}{14}$ (A1) for substitution

$\mathrm{E}(X)=\frac{18}{7}$ A1 N3

[3]

(c) $\mathrm{P}(Y=3)=\left(\frac{1}{14}\right)\left(\frac{4}{14}\right)+\left(\frac{4}{14}\right)\left(\frac{1}{14}\right)$ (A1) for correct formula

$\mathrm{P}(Y=3)=\frac{2}{49}$ A1 N2

[2]

4. (a) $\mathrm{P}(X=k)+\mathrm{P}(X=k+1)+\mathrm{P}(X=k+2)+\mathrm{P}(X=8)=1$ (A1) for correct formula

$\frac{k}{2}+\frac{1}{8}+\frac{k}{4}+\frac{1}{8}=1$

$\frac{3k}{4}+\frac{1}{4}=1$ (A1) for simplification

$3k+1=4$

$k=1$ A1 N3

[3]

(b) $\mathrm{E}(X)=\frac{k}{2}(k)+\frac{1}{8}(k+1)+\frac{k}{4}(k+2)+\frac{1}{8}(8)$ (A1) for correct formula

$\mathrm{E}(X)=\frac{1}{2}+\frac{2}{8}+\frac{3}{4}+1$ (A1) for substitution

$\mathrm{E}(X)=\frac{5}{2}$ A1 N3

[3]

(c) $\mathrm{P}(Y=2)=\left(\frac{1}{2}\right)\left(\frac{1}{8}\right)+\left(\frac{1}{8}\right)\left(\frac{1}{2}\right)$ (A1) for correct formula

$\mathrm{P}(Y=2)=\frac{1}{8}$ A1 N2

[2]

Exercise 78

1. (a) $\mathrm{P}(X=4)+\mathrm{P}(X=8)+\mathrm{P}(X=12)=1$

$10k^2+k+20k^2=1$ (A1) for substitution

$30k^2+k-1=0$

$(6k-1)(5k+1)=0$ (A1) for factorization

$k=\frac{1}{6}$ or $k=-\frac{1}{5}$ (*Rejected*) A1 N3

[3]

(b) $\mathrm{P}(X=12\,|\,X>6)=\frac{\mathrm{P}(X=12\cap X>6)}{\mathrm{P}(X>6)}$ (M1) for valid approach

$\mathrm{P}(X=12\,|\,X>6)=\frac{\mathrm{P}(X=12)}{\mathrm{P}(X>6)}$

$\mathrm{P}(X=12\,|\,X>6)=\frac{20\left(\frac{1}{6}\right)^2}{20\left(\frac{1}{6}\right)^2+\frac{1}{6}}$ (A1) for substitution

$\mathrm{P}(X=12\,|\,X>6)=\frac{10}{13}$ A1 N3

[3]

2. (a) $\mathrm{P}(X=12)+\mathrm{P}(X=24)+\mathrm{P}(X=30)+\mathrm{P}(X=36)=1$

$k+7k^2+8k^2+k=1$ (A1) for substitution

$15k^2+2k-1=0$

$(5k-1)(3k+1)=0$ (A1) for factorization

$k=\frac{1}{5}$ or $k=-\frac{1}{3}$ (*Rejected*) A1 N3

[3]

(b) $\mathrm{P}(X=24\,|\,X>20)=\frac{\mathrm{P}(X=24\cap X>20)}{\mathrm{P}(X>20)}$ (M1) for valid approach

$\mathrm{P}(X=24\,|\,X>20)=\frac{\mathrm{P}(X=24)}{\mathrm{P}(X>20)}$

$\mathrm{P}(X=24\,|\,X>20)=\frac{7\left(\frac{1}{5}\right)^2}{7\left(\frac{1}{5}\right)^2+8\left(\frac{1}{5}\right)^2+\frac{1}{5}}$ (A1) for substitution

$\mathrm{P}(X=24\,|\,X>20)=\frac{7}{20}$ A1 N3

[3]

3. (a)

$$P(X=7)+P(X=14)+P(X=21)$$
$$+P(X=28)+P(X=35)=1$$
$$k+3k+10k^2+6k^2+5k^2=1 \quad \text{(A1) for substitution}$$
$$21k^2+4k-1=0$$
$$(7k-1)(3k+1)=0 \quad \text{(A1) for factorization}$$
$$k=\frac{1}{7} \text{ or } k=-\frac{1}{3} \text{ (}\textit{Rejected}\text{)} \quad \text{A1 N3}$$

[3]

(b) $P(X<15 \mid X<25)=\dfrac{P(X<15\cap X<25)}{P(X<25)}$ (M1) for valid approach

$$P(X<15 \mid X<25)=\frac{P(X<15)}{P(X<25)}$$

$$P(X<15 \mid X<25)=\frac{\frac{1}{7}+3\left(\frac{1}{7}\right)}{\frac{1}{7}+3\left(\frac{1}{7}\right)+10\left(\frac{1}{7}\right)^2} \quad \text{(A1) for substitution}$$

$$P(X<15 \mid X<25)=\frac{14}{19} \quad \text{A1 N3}$$

[3]

4. (a)

$$P(X=0)+P(X=1)+P(X=2)$$
$$+P(X=3)+P(X=4)+P(X=5)=1$$
$$k^2+k+4k^2+8k^2+4k+k^2=1 \quad \text{(A1) for substitution}$$
$$14k^2+5k-1=0$$
$$(7k-1)(2k+1)=0 \quad \text{(A1) for factorization}$$
$$k=\frac{1}{7} \text{ or } k=-\frac{1}{2} \text{ (}\textit{Rejected}\text{)} \quad \text{A1 N3}$$

[3]

(b) $P(2<X\le 4 \mid 1<X\le 4)=\dfrac{P(2<X\le 4\cap 1<X\le 4)}{P(1<X\le 4)}$ (M1) for valid approach

$$P(2<X\le 4 \mid 1<X\le 4)=\frac{P(2<X\le 4)}{P(1<X\le 4)}$$

$$P(2<X\le 4 \mid 1<X\le 4)=\frac{8\left(\frac{1}{7}\right)^2+4\left(\frac{1}{7}\right)}{4\left(\frac{1}{7}\right)^2+8\left(\frac{1}{7}\right)^2+4\left(\frac{1}{7}\right)} \quad \text{(A1) for substitution}$$

$$P(2<X\le 4 \mid 1<X\le 4)=\frac{9}{10} \quad \text{A1 N3}$$

[3]

Exercise 79

1. (a) $\mathrm{P}(X=1)+\mathrm{P}(X=2)+\mathrm{P}(X=3)+\mathrm{P}(X=4)=1$ (A1) for correct formula

$0.2+0.3+a+b=1$

$a+b=0.5$ A1 N2

[2]

(b) $\mathrm{E}(X)=2.62$

$0.2(1)+0.3(2)+3a+4b=2.62$ (A1) for correct formula

$3a+4b=1.82$ A1 N2

[2]

(c) $a=0.18,\ b=0.32$ A2 N2

[2]

2. (a) $\mathrm{P}(X<45)=0.9$

$\mathrm{P}(X=20)+\mathrm{P}(X=30)+\mathrm{P}(X=40)=0.9$ (A1) for correct formula

$0.1+a+b=0.9$

$a+b=0.8$ A1 N2

[2]

(b) $\mathrm{E}(X)=33$

$0.1(20)+30a+40b+0.1(50)=33$ (A1) for correct formula

$30a+40b=26$ A1 N2

[2]

(c) $a=0.6,\ b=0.2$ A2 N2

[2]

3. (a) $\mathrm{P}(X < 15) = 0.5$

$\mathrm{P}(X = 0) + \mathrm{P}(X = 10) = 0.5$ (A1) for correct formula

$0.1 + a = 0.5$

$a = 0.4$ A1 N2

[2]

(b) $\mathrm{P}(X = 0) + \mathrm{P}(X = 10) + \mathrm{P}(X = 20) + \mathrm{P}(X = 30) = 1$

$0.1 + 0.4 + b + c = 1$ (A1) for correct formula

$b + c = 0.5$

$\mathrm{E}(X) = 16$

$0.1(0) + 10(0.4) + 20b + 30c = 16$ (A1) for correct formula

$20b + 30c = 12$

By solving the system $\begin{cases} b + c = 0.5 \\ 20b + 30c = 12 \end{cases}$, $b = 0.3$

and $c = 0.2$. A2 N4

[4]

(c) $\mathrm{P}(Y = 50) = (0.3)(0.2) + (0.2)(0.3)$ (A1) for correct formula

$\mathrm{P}(Y = 50) = 0.12$ A1 N2

[2]

4. (a) $\mathrm{P}(X = 0) + \mathrm{P}(X = 3) + \mathrm{P}(X = 6) + \mathrm{P}(X = 9) = 1$

$0.4 + 0.3 + c = 1$ (A1) for correct formula

$c = 0.3$ A1 N2

[2]

(b) $\mathrm{P}(2 < X < 7) = 0.3$

$\mathrm{P}(X = 3) + \mathrm{P}(X = 6) = 0.3$ (A1) for correct formula

$a + b = 0.3$

$\mathrm{E}(X) = 4.2$

$0.4(0) + 3a + 6b + 9(0.3) = 4.2$ (A1) for correct formula

$3a + 6b = 1.5$

By solving the system $\begin{cases} a + b = 0.3 \\ 3a + 6b = 1.5 \end{cases}$, $a = 0.1$

and $b = 0.2$. A2 N4

[4]

(c) $\mathrm{P}(Y = 36) = (0.2)(0.2)$ (A1) for correct formula

$\mathrm{P}(Y = 36) = 0.04$ A1 N2

[2]

Exercise 80

1. (a) (i) $P(F \cap S) = (0.6)(0.6)$ (A1) for substitution

$P(F \cap S) = 0.36$ A1 N2

(ii) $P(S) = P(F \cap S) + P(F' \cap S)$ (M1) for valid approach

$P(S) = 0.36 + (0.4)(0.6)$

$P(S) = 0.6$ A1 N2

[4]

(b) (i) The required probability

$= P(F' \cap S')$

$= (0.4)(0.4)$ (A1) for substitution

$= 0.16$ A1 N2

(ii) The required probability

$= P(F \mid S')$ (M1) for valid approach

$= \dfrac{P(F \cap S')}{P(S')}$

$= \dfrac{(0.6)(0.4)}{1-0.6}$ (A1) for substitution

$= 0.6$ A1 N3

[5]

(c)

X	2	5	8
$P(X = x)$	0.16	0.48	0.36

A3 N3

[3]

(d) The expected value

$= (2)(0.16) + (5)(0.48) + (8)(0.36)$ (A1) for substitution

$= 5.6$ A1 N2

[2]

2. (a) (i) $\mathrm{P}(S \cap L') = (0.2)(0.3)$ (A1) for substitution

$\mathrm{P}(S \cap L') = 0.06$ A1 N2

(ii) $\mathrm{P}(L') = \mathrm{P}(S \cap L') + \mathrm{P}(S' \cap L')$ (M1) for valid approach

$\mathrm{P}(L') = 0.06 + (0.8)(0.6)$

$\mathrm{P}(L') = 0.54$ A1 N2

[4]

(b) (i) The required probability

$= \mathrm{P}(S' \cap L')$

$= (0.8)(0.6)$ (A1) for substitution

$= 0.48$ A1 N2

(ii) The required probability

$= \mathrm{P}(S \mid L)$ (M1) for valid approach

$= \dfrac{\mathrm{P}(S \cap L)}{\mathrm{P}(L)}$

$= \dfrac{(0.2)(0.7)}{1 - 0.54}$ (A1) for substitution

$= \dfrac{7}{23}$ A1 N3

[5]

(c)

X	0	10	25
$\mathrm{P}(X = x)$	0.09	0.42	0.49

A3 N3

[3]

(d) The expected value

$= (0)(0.09) + (10)(0.42) + (25)(0.49)$ (A1) for substitution

$= 16.45$ A1 N2

[2]

3. (a) (i) $\mathrm{P}(T' \cap L') = \left(1-\frac{1}{2}\right)\left(1-\frac{3}{10}\right)$ (A1) for substitution

$\mathrm{P}(T' \cap L') = \frac{7}{20}$ A1 N2

(ii) $\mathrm{P}(L') = \mathrm{P}(T \cap L') + \mathrm{P}(T' \cap L')$ (M1) for valid approach

$\mathrm{P}(L') = \left(\frac{1}{2}\right)\left(1-\frac{9}{10}\right)+\frac{7}{20}$

$\mathrm{P}(L') = \frac{2}{5}$ A1 N2

[4]

(b) (i) The required probability

$= \mathrm{P}(T \cap L')$

$= \left(\frac{1}{2}\right)\left(1-\frac{9}{10}\right)$ (A1) for substitution

$= \frac{1}{20}$ A1 N2

(ii) The required probability

$= \mathrm{P}(L' \mid T')$ (M1) for valid approach

$= \frac{\mathrm{P}(L' \cap T')}{\mathrm{P}(T')}$

$= \frac{\frac{7}{20}}{1-\frac{1}{2}}$ (A1) for substitution

$= \frac{7}{10}$ A1 N3

[5]

(c)

X	0	125	250	375
$\mathrm{P}(X = x)$	$\frac{8}{125}$	$\frac{36}{125}$	$\frac{54}{125}$	$\frac{27}{125}$

A3 N3

[3]

(d) The expected expenditure

$$= (0)\left(\frac{8}{125}\right) + (125)\left(\frac{36}{125}\right) + (250)\left(\frac{54}{125}\right)$$

$$+(375)\left(\frac{27}{125}\right)$$

$$= \$225$$

(A1) for substitution

A1 N2

[2]

4. (a) (i) $\mathrm{P}(R' \cap A) = (1-0.5)(0.4)$ (A1) for substitution

$\mathrm{P}(R' \cap A) = 0.2$ A1 N2

(ii) $\mathrm{P}(A) = \mathrm{P}(R \cap A) + \mathrm{P}(R' \cap A)$ (M1) for valid approach

$\mathrm{P}(A) = (0.5)(0.8) + 0.2$

$\mathrm{P}(A) = 0.6$ A1 N2

[4]

(b) (i) The required probability

$= \mathrm{P}(R \cap A')$

$= (0.5)(1-0.8)$ (A1) for substitution

$= 0.1$ A1 N2

(ii) The required probability

$= \mathrm{P}(R \mid A)$ (M1) for valid approach

$= \dfrac{\mathrm{P}(R \cap A)}{\mathrm{P}(A)}$

$= \dfrac{(0.5)(0.8)}{0.6}$ (A1) for substitution

$= \dfrac{2}{3}$ A1 N3

[5]

(c)

X	0	4	8	12
$\mathrm{P}(X = x)$	$\frac{27}{125}$	$\frac{54}{125}$	$\frac{36}{125}$	$\frac{8}{125}$

A3 N3

[3]

(d) The expected expenditure

$= (0)\left(\dfrac{27}{125}\right) + (4)\left(\dfrac{54}{125}\right) + (8)\left(\dfrac{36}{125}\right) + (12)\left(\dfrac{8}{125}\right)$ (A1) for substitution

$= \$4.8$ A1 N2

[2]

Exercise 81

1. (a) (i) There are 4 ways such that $X = 5$ — (A1) for correct value

$$P(X = 5) = \frac{4}{36}$$

$$P(X = 5) = \frac{1}{9}$$ A1 N2

(ii) There are 6 ways such that $X < 5$ — (A1) for correct value

$$P(X < 5) = \frac{6}{36}$$

$$P(X < 5) = \frac{1}{6}$$ A1 N2

(iii) $$P(X = 4 \mid X < 6) = \frac{P(X = 4 \cap X < 6)}{P(X < 6)}$$ (M1) for valid approach

$$P(X = 4 \mid X < 6) = \frac{P(X = 4)}{P(X < 6)}$$

$$P(X = 4 \mid X < 6) = \frac{\frac{3}{36}}{\frac{1}{9} + \frac{1}{6}}$$ (A1) for substitution

$$P(X = 4 \mid X < 6) = \frac{3}{10}$$ A1 N3

[7]

(b) (i) $\frac{13}{18}$ A1 N1

(ii) $E(X) = 0$

$(3)P(X = 5) + (2)P(X < 5)$
$+(-k)P(X > 5) = 0$ — (M1)(A1) for correct formula

$$(3)\left(\frac{1}{9}\right) + (2)\left(\frac{1}{6}\right) + (-k)\left(\frac{13}{18}\right) = 0$$ (A1) for substitution

$$6 + 6 - 13k = 0$$

$$k = \frac{12}{13}$$ A1 N4

[5]

2. (a) (i) There are 5 ways such that $X = 8$ (A1) for correct value

$$P(X = 8) = \frac{5}{36}$$ A1 N2

(ii) There are 10 ways such that $X > 8$ (A1) for correct value

$$P(X > 8) = \frac{10}{36}$$

$$P(X > 8) = \frac{5}{18}$$ A1 N2

(iii) $$P(X > 9 \mid X > 8) = \frac{P(X > 9 \cap X > 8)}{P(X > 8)}$$ (M1) for valid approach

$$P(X > 9 \mid X > 8) = \frac{P(X > 9)}{P(X > 8)}$$

$$P(X > 9 \mid X > 8) = \frac{\frac{6}{36}}{\frac{5}{18}}$$ (A1) for substitution

$$P(X > 9 \mid X > 8) = \frac{3}{5}$$ A1 N3

[7]

(b) (i) $\frac{7}{12}$ A1 N1

(ii) $E(X) = 1$

$(5)P(X = 8) + (k)P(X > 8)$

$+(-1)P(X < 8) = 1$ (M1)(A1) for correct formula

$$(5)\left(\frac{5}{36}\right) + (k)\left(\frac{5}{18}\right) + (-1)\left(\frac{7}{12}\right) = 1$$ (A1) for substitution

$25 + 10k - 21 = 36$

$k = 3.2$ A1 N4

[5]

3. (a) (i) There is only 1 way such that $X = 21$ (A1) for correct value

$$P(X = 21) = \frac{1}{9}$$ A1 N2

(ii) There are 5 ways such that $X > 21$ (A1) for correct value

$$P(X > 21) = \frac{5}{9}$$ A1 N2

(iii) $P(30 < X < 33 \mid X > 21)$

$$= \frac{P(30 < X < 33 \cap X > 21)}{P(X > 21)}$$ (M1) for valid approach

$$= \frac{P(30 < X < 33)}{P(X > 21)}$$

$$= \frac{\frac{2}{9}}{\frac{5}{9}}$$ (A1) for substitution

$$= \frac{2}{5}$$ A1 N3

[7]

(b) $P(X < 21) = 1 - P(X = 21) - P(X > 21)$

$$P(X < 21) = 1 - \frac{1}{9} - \frac{5}{9}$$ (A1) for substitution

$$P(X < 21) = \frac{1}{3}$$ (A1) for correct value

$E(X) = 8$

$(3k)P(X = 21) + (k)P(X > 21) + (0)P(X < 21) = 8$ (M1)(A1) for correct formula

$$\therefore (3k)\left(\frac{1}{9}\right) + (k)\left(\frac{5}{9}\right) + (0)\left(\frac{1}{3}\right) = 8$$ (A1) for substitution

$3k + 5k = 72$

$k = 9$ A1 N6

[6]

4. (a) (i) There is only 1 way such that $X = 33$ (A1) for correct value

$$P(X = 33) = \frac{1}{9}$$ A1 N2

(ii) There are 2 ways such that $X \geq 35$ (A1) for correct value

$$P(X \geq 35) = \frac{2}{9}$$ A1 N2

(iii) $$P(X < 22 \mid X < 33) = \frac{P(X < 22 \cap X < 33)}{P(X < 33)}$$ (M1) for valid approach

$$P(X < 22 \mid X < 33) = \frac{P(X < 22)}{P(X < 33)}$$

$$P(X < 22 \mid X < 33) = \frac{\frac{5}{9}}{\frac{6}{9}}$$ (A1) for substitution

$$P(X < 22 \mid X < 33) = \frac{5}{6}$$ A1 N3

[7]

(b) $$P(X < 33) = 1 - P(X = 33) - P(X > 33)$$

$$P(X < 33) = 1 - \frac{1}{9} - \frac{2}{9}$$ (A1) for substitution

$$P(X < 33) = \frac{2}{3}$$ (A1) for correct value

$$E(X) = -16$$

$$(4k)P(X = 33) + (3k)P(X > 33) + (-2k)P(X < 33) = -16$$ (M1)(A1) for correct formula

$$\therefore (4k)\left(\frac{1}{9}\right) + (3k)\left(\frac{2}{9}\right) + (-2k)\left(\frac{2}{3}\right) = -16$$ (A1) for substitution

$$4k + 6k - 12k = -144$$

$$k = 72$$ A1 N6

[6]

Chapter 20 Solution

Exercise 82

1. (a) The expected number

$= (10)(0.25)$ (A1) for substitution

$= 2.5$ A1 N2

[2]

(b) The variance

$= (10)(0.25)(1-0.25)$ (A1) for substitution

$= 1.875$ A1 N2

[2]

(c) The required probability

$= \binom{10}{3}(0.25)^3(1-0.25)^{10-3}$ (A1) for substitution

$= 0.250282287$

$= 0.250$ A1 N2

[2]

2. (a) $\mathrm{E}(X) = 18$

$0.6n = 18$ (A1) for substitution

$n = 30$ A1 N2

[2]

(b) $\mathrm{Var}(X)$

$= (30)(0.6)(1-0.6)$ (A1) for substitution

$= 7.2$ A1 N2

[2]

(c) $\mathrm{P}(X = 19)$

$= \binom{30}{19}(0.6)^{19}(1-0.6)^{30-19}$ (A1) for substitution

$= 0.139618638$

$= 0.140$ A1 N2

[2]

3. (a) $20p(1-p)=3.2$ (A1) for substitution

$-20p^2+20p-3.2=0$ (M1) for quadratic equation

By considering the graph of $y=-20p^2+20p-3.2$,

$p=0.8$ or $p=0.2$ (*Rejected*). A1 N3

[3]

(b) The expected number

$=(20)(0.8)$ (A1) for substitution

$=16$ A1 N2

[2]

(c) The required probability

$=\binom{20}{17}(0.8)^{17}(1-0.8)^{20-17}$ (A1) for substitution

$=0.205364143$

$=0.205$ A1 N2

[2]

4. (a) $\begin{cases} np=1800 \\ np(1-p)=990 \end{cases}$ (A1) for correct equations

$1800(1-p)=990$ (M1) for substitution

$1-p=0.55$

$p=0.45$ A1 N3

[3]

(b) $0.45n=1800$ (A1) for substitution

$n=4000$ A1 N2

[2]

(c) 2.3511×10^{-11} A2 N2

[2]

Exercise 83

1. (a) $\mathrm{E}(X) = (80)(0.06)$ (A1) for substitution

$\mathrm{E}(X) = 4.8$ A1 N2

[2]

(b) $\mathrm{P}(X = 10)$

$= \binom{80}{10}(0.06)^{10}(1-0.06)^{80-10}$ (A1) for substitution

$= 0.0130924797$

$= 0.0131$ A1 N2

[2]

(c) $\mathrm{P}(X \geq 15)$

$= 1 - \mathrm{P}(X \leq 14)$ (M1) for valid approach

$= 1 - 0.9999251314$ (A1) for correct value

$= 0.0000748686$

$= 0.0000749$ A1 N3

[3]

2. (a) $\mathrm{E}(X) = (135)(0.12)$ (A1) for substitution

$\mathrm{E}(X) = 16.2$ A1 N2

[2]

(b) $\mathrm{P}(X = 20)$

$= \binom{135}{20}(0.12)^{20}(1-0.12)^{135-20}$ (A1) for substitution

$= 0.0597993427$

$= 0.0598$ A1 N2

[2]

(c) $\mathrm{P}(X > 16)$

$= 1 - \mathrm{P}(X \leq 16)$ (M1) for valid approach

$= 1 - 0.5449524887$ (A1) for correct value

$= 0.4550475113$

$= 0.455$ A1 N3

[3]

3. (a) $\mathrm{E}(X) = (50)(0.02)$ (A1) for substitution

$\mathrm{E}(X) = 1$ A1 N2

[2]

(b) $\mathrm{P}(X = 9)$

$= \binom{50}{9}(0.02)^{9}(1-0.02)^{50-9}$ (A1) for substitution

$= 0.000000560302$

$= 0.000000560$ A1 N2

[2]

(c) $\mathrm{P}(X \leq 2)$

$= 0.9215722517$ (M1) for valid approach

$= 0.922$ A1 N2

[2]

4. (a) The mean number of heads

$= (9)(0.69)$ (A1) for substitution

$= 6.21$ A1 N2

[2]

(b) The required probability

$= \binom{9}{6}(0.69)^{6}(1-0.69)^{9-6}$ (A1) for substitution

$= 0.2700591597$

$= 0.270$ A1 N2

[2]

(c) The required probability

$= 0.005271637$ (M1) for valid approach

$= 0.00527$ A1 N2

[2]

Exercise 84

1. (a) The required probability

$= \binom{120}{3} p^3(1-p)^{120-3}$ (A1) for substitution

$= \binom{120}{3} p^3(1-p)^{117}$ A1 N2

[2]

(b) $\binom{120}{3} p^3(1-p)^{117} = 0.16$ (M1) for setting equation

$\binom{120}{3} p^3(1-p)^{117} - 0.16 = 0$

By considering the graph of

$y = \binom{120}{3} p^3(1-p)^{117} - 0.16$, $p = 0.0148695$

or $p = 0.0388023$.

$\therefore p = 0.0149$ or $p = 0.0388$ A2 N3

[3]

2. (a) The required probability

$= \binom{5}{4} p^4(1-p)^{5-4}$ (A1) for substitution

$= 5p^4(1-p)$ A1 N2

[2]

(b) $5p^4(1-p) = 0.3$ (M1) for setting equation

$5p^4(1-p) - 0.3 = 0$

By considering the graph of $y = 5p^4(1-p) - 0.3$,

$p = 0.6381051$ or $p = 0.9140419$.

$\therefore p = 0.638$ or $p = 0.914$ A2 N3

[3]

3. (a) The required probability

$= \binom{10}{9} q^9 (1-q)^{10-9} + \binom{10}{10} q^{10} (1-q)^{10-10}$ (A1) for substitution

$= 10q^9(1-q) + q^{10}$ A1 N2

[2]

(b) $10q^9(1-q) + q^{10} = 0.09$ (M1) for setting equation

$10q^9(1-q) + q^{10} - 0.09 = 0$

By considering the graph of

$y = 10q^9(1-q) + q^{10} - 0.09$, $q = 0.6539559$.

$\therefore q = 0.654$ A2 N3

[3]

4. (a) The required probability

$= \binom{100}{0} q^0 (1-q)^{100-0} + \binom{100}{1} q^1 (1-q)^{100-1}$ (A1) for substitution

$= (1-q)^{100} + 100q(1-q)^{99}$ A1 N2

[2]

(b) $(1-q)^{100} + 100q(1-q)^{99} = 0.03$ (M1) for setting equation

$(1-q)^{100} + 100q(1-q)^{99} - 0.03 = 0$

By considering the graph of

$y = (1-q)^{100} + 100q(1-q)^{99} - 0.03$, $q = 0.0524073$.

$\therefore q = 0.0524$ A2 N3

[3]

Exercise 85

1. (a) The required probability

$= 0.56 \times 0.12 + (1 - 0.56) \times 0.76$ (M1)(A1) for substitution

$= 0.56 \times 0.12 + 0.44 \times 0.76$

$= 0.4016$ A1 N3

[3]

(b) The required probability

$= \dfrac{0.44 \times 0.76}{0.4016}$ (M1)(A1) for substitution

$= 0.8326693227$

$= 0.833$ A1 N3

[3]

(c) $X \sim B(6, 0.5984)$ (R1) for binomial distribution

$\mathrm{P}(X = 4)$

$= \binom{6}{4} (0.5984)^4 (1 - 0.5984)^{6-4}$ (A1) for substitution

$= 0.3102022951$

$= 0.310$ A1 N3

[3]

(d) The probability that Joyce did not stay at home for all n days is 0.4016^n. (M1) for valid approach

$1 - 0.4016^n > 0.84$ (M1)(A1) for correct inequality

$0.16 - 0.4016^n > 0$

By considering the graph of $y = 0.16 - 0.4016^n$,

$n > 2.0087516$. (A1) for correct value

$\therefore n = 3$ A1 N5

[5]

2. (a) The required probability

$= 0.4 \times 0.2 + (1 - 0.4) \times 0.3$ (M1)(A1) for substitution

$= 0.4 \times 0.2 + 0.6 \times 0.3$

$= 0.26$ A1 N3

[3]

(b) The required probability

$= \dfrac{0.6 \times 0.3}{0.26}$ (M1)(A1) for substitution

$= 0.6923076923$

$= 0.692$ A1 N3

[3]

(c) $X \sim B(4, 0.26)$ (R1) for binomial distribution

$\mathrm{P}(X = 2)$

$= \binom{4}{2}(0.26)^2(1 - 0.26)^{4-2}$ (A1) for substitution

$= 0.22210656$

$= 0.222$ A1 N3

[3]

(d) $1 - 0.74^n - n(0.74)^{n-1}(0.26) > 0.75$ (M1)(A1) for correct inequality

$0.25 - 0.74^n - 0.26n(0.74)^{n-1} > 0$ (M1) for simplification

By considering the graph of

$y = 0.25 - 0.74^n - 0.26n(0.74)^{n-1}, n > 9.4689646$. (A1) for correct value

$\therefore n = 10$ A1 N5

[5]

3. (a) The required probability

$= 0.45 \times 0.13 + (1 - 0.45) \times 0.59$ (M1)(A1) for substitution

$= 0.45 \times 0.13 + 0.55 \times 0.59$

$= 0.383$ A1 N3

[3]

(b) The required probability

$= \dfrac{0.55 \times 0.59}{0.383}$ (M1)(A1) for substitution

$= 0.8472584856$

$= 0.847$ A1 N3

[3]

(c) $X \sim B(7, 0.383)$ (R1) for binomial distribution

$\mathrm{P}(X = 3)$

$= \binom{7}{3} (0.383)^3 (1 - 0.383)^{7-3}$ (A1) for substitution

$= 0.2849738583$

$= 0.285$ A1 N3

[3]

(d) The probability that Lydia caught a fish at most one day is $(1 - 0.383)^n + n(1 - 0.383)^{n-1}(0.383)$. (M1) for valid approach

$1 - [(1 - 0.383)^n + n(1 - 0.383)^{n-1}(0.383)] > 0.93$ (M1)(A1) for correct inequality

$0.07 - 0.617^n - 0.383n \cdot 0.617^{n-1} > 0$

By considering the graph of

$y = 0.07 - 0.617^n - 0.383n \cdot 0.617^{n-1}$,

$n > 9.5074803$. (A1) for correct value

$\therefore n = 10$ A1 N5

[5]

4. (a) The required probability

$= p \times 0.3 + (1-p) \times 0.48$ (M1)(A1) for substitution

$= 0.3p + 0.48 - 0.48p$

$= 0.48 - 0.18p$ A1 N3

[3]

(b) $\dfrac{0.3p}{0.48 - 0.18p}$ A2 N2

[2]

(c) $X \sim B(8, 0.3702)$ (R1) for binomial distribution

$\mathrm{P}(X = 6)$

$= \binom{8}{6}(0.3702)^6(1-0.3702)^{8-6}$ (A1) for substitution

$= 0.0285878721$

$= 0.0286$ A1 N3

[3]

(d) The probability that reaching the escape door for at most two trial

$= (1-0.3702)^n + n(1-0.3702)^{n-1}(0.3702)$

$+ \binom{n}{2}(1-0.3702)^{n-2}(0.3702)^2$ (M1) for valid approach

$1 - [0.6298^n + n(0.6298)^{n-1}(0.3702)$

$+ \binom{n}{2}(0.6298)^{n-2}(0.3702)^2] > 0.99$ (M1)(A1) for correct inequality

$0.01 - 0.6298^n - 0.3702n(0.6298)^{n-1}$

$- \binom{n}{2}(0.6298)^{n-2}(0.3702)^2 > 0$

By considering the graph of

$y = 0.01 - 0.6298^n - 0.3702n(0.6298)^{n-1}$

$- \left(\dfrac{n(n-1)}{2}\right)(0.6298)^{n-2}(0.3702)^2$,

$n > 19.237508.$ (A1) for correct value

$\therefore n = 20$ A1 N5

[5]

Chapter 21 Solution

Exercise 86

1. (a) $P(X > 86) = 0.28$ A1 N1

[1]

(b) $P(80 < X < 86) = P(X > 80) - P(X > 86)$ (M1) for valid approach

$P(80 < X < 86) = 0.5 - 0.28$ (A1) for substitution

$P(80 < X < 86) = 0.22$ A1 N3

[3]

(c) $P(74 < X < 80) = P(80 < X < 86)$ (M1) for symmetric property

$P(74 < X < 80) = 0.22$ A1 N2

[2]

2. (a) $P(X < 270) = 0.15$ A1 N1

[1]

(b) $P(270 < X < 300) = P(X < 300) - P(X < 270)$ (M1) for valid approach

$P(270 < X < 300) = 0.5 - 0.15$ (A1) for substitution

$P(270 < X < 300) = 0.35$ A1 N3

[3]

(c) $P(270 < X < 330) = 2 \times P(270 < X < 300)$ (M1) for symmetric property

$P(270 < X < 330) = 0.7$ A1 N2

[2]

3. (a) $P(X > 2.7) = 0.07$ A1 N1

[1]

(b) $P(1.5 < X < 2.7) = P(X > 1.5) - P(X > 2.7)$ (M1) for valid approach

$P(1.5 < X < 2.7) = 0.5 - 0.07$ (A1) for substitution

$P(1.5 < X < 2.7) = 0.43$ A1 N3

[3]

(c) $P(X > 0.3) = 2 \times P(1.5 < X < 2.7) + P(X > 2.7)$ (M1) for symmetric property

$P(X > 0.3) = 0.93$ A1 N2

[2]

4. (a) $\mathrm{P}\left(X > \frac{3}{11}\right) = 1 - \frac{1}{6}$ (M1) for valid approach

$\mathrm{P}\left(X > \frac{3}{11}\right) = \frac{5}{6}$ A1 N2

[2]

(b) $d - \frac{6}{11} = \frac{6}{11} - \frac{3}{11}$ (M1) for valid approach

$d = \frac{9}{11}$ A1 N2

[2]

(c) $\mathrm{P}\left(\frac{3}{11} < X < d\right) = 1 - 2 \times \mathrm{P}\left(X < \frac{3}{11}\right)$ (M1) for symmetric property

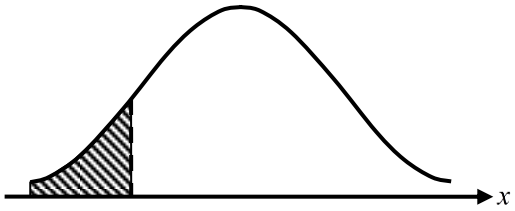
$\mathrm{P}\left(\frac{3}{11} < X < d\right) = \frac{2}{3}$ A1 N2

[2]

Exercise 87

1. (a) For vertical line clearly to the left of the mean A1
For shading to the left of the vertical line A1 N2

[2]



(b) $P(X \leq 60) = 0.022750062$ (A1) for correct value
$P(X \leq 60) = 0.0228$ A1 N2

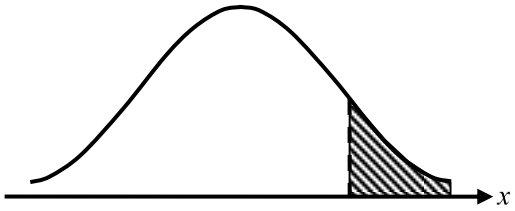
[2]

(c) $c = 61.7961209$
$c = 61.8$ A2 N2

[2]

2. (a) For vertical line clearly to the right of the mean A1
For shading to the right of the vertical line A1 N2

[2]



(b) $P(X \geq 4.83) = 0.2653735838$ (A1) for correct value
$P(X \geq 4.83) = 0.2654$ A1 N2

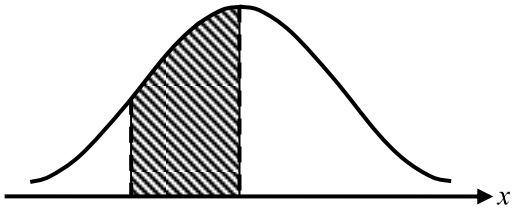
[2]

(c) $c = 4.7613483$
$c = 4.76$ A2 N2

[2]

3. (a) For vertical lines clearly to the left of the mean and at the mean — A1
For shading area bounded by the vertical lines — A1 N2

[2]



(b) $P(23.5 \le X \le 30) = 0.4479187309$ — (A1) for correct value
$P(23.5 \le X \le 30) = 0.4479$ — A1 N2

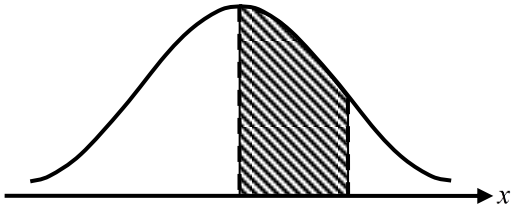
[2]

(c) $c = 32.0976017$
$c = 32.1$ — A2 N2

[2]

4. (a) For vertical lines clearly at the mean and to the right of the mean — A1
For shading area bounded by the vertical lines — A1 N2

[2]



(b) $P(162 \le X \le 171) = 0.3697054352$ — (A1) for correct value
$P(162 \le X \le 171) = 0.3697$ — A1 N2

[2]

(c) $c = 158.0331972$
$c = 158$ — A2 N2

[2]

Exercise 88

1. (a) $P(X < Q_1) = 0.25$ (M1) for valid approach

$Q_1 = 229.882656$

Thus, the lower quartile is 230. A1 N2

[2]

(b) $P(X < Q_3) = 0.75$ (M1) for valid approach

$Q_3 = 250.117344$

Thus, the upper quartile is 250. A1 N2

[2]

(c) The interquartile range of X

$= Q_3 - Q_1$ (A1) for correct formula

$= 250.117344 - 229.882656$

$= 20.234688$

$= 20.2$ A1 N2

[2]

2. (a) $P(X < q_{30}) = 0.3$ (M1) for valid approach

$q_{30} = 155.804797$

Thus, the 30th percentile is 156. A1 N2

[2]

(b) $P(X < q_{70}) = 0.7$ (M1) for valid approach

$q_{70} = 164.195204$

Thus, the 70th percentile is 164. A1 N2

[2]

(c) $q_{70} - q_{30}$

$= 164.195204 - 155.804797$ (A1) for substitution

$= 8.390407$

$= 8.39$ A1 N2

[2]

3.

$\mathrm{P}(X < q_{90}) = 0.9$ (M1) for valid approach

$q_{90} = 93.126207$ (A1) for correct value

$\mathrm{P}(X < q_{10}) = 0.1$ (M1) for valid approach

$q_{10} = 82.87379373$ (A1) for correct value

$q_{90} - q_{10}$ (A1) for correct formula

$= 93.126207 - 82.87379373$

$= 10.25241327$

$= 10.3$ A1 N6

[6]

4.

$\mathrm{P}(X < s) = \frac{1}{3}$ (M1) for valid approach

$s = 48.707818$ (A1) for correct value

$\mathrm{P}(X < t) = \frac{2}{3}$ (M1) for valid approach

$t = 51.292182$ (A1) for correct value

$t - s$

$= 51.292182 - 48.707818$ (A1) for substitution

$= 2.584364$

$= 2.58$ A1 N6

[6]

Exercise 89

1. (a) (i) Let W be the weight of a randomly selected fish.

The required probability

$= \mathrm{P}(W > 850)$ (M1) for valid approach

$= 0.0083943057$

$= 0.00839$ A1 N2

(ii) $\mathrm{P}(W > 900 \mid W > 850)$ (R1) for correct probability

$= \dfrac{\mathrm{P}(W > 850 \cap W > 900)}{\mathrm{P}(W > 850)}$ (A1) for correct formula

$= \dfrac{\mathrm{P}(W > 900)}{\mathrm{P}(W > 850)}$

$= \dfrac{0.000252385136}{0.0083943057}$ (A1) for correct values

$= 0.0300662312$

$= 0.0301$ A1 N4

[6]

(b) The required probability

$= \mathrm{P}(W > 850) \times \mathrm{P}(W > 850)$ (M1) for valid approach

$= 0.0083943057 \times 0.0083943057$

$= 0.00007046436819$

$= 0.0000705$ A1 N2

[2]

(c) (i) The required expected number

$= 0.0083943057 \times 100$ (A1) for correct formula

$= 0.83943057$

$= 0.839$ A1 N2

(ii) Let X : Number of big fish in the selected sample

$X \sim B(100, 0.0083943057)$ (R1) for binomial distribution

The required probability

$= \mathrm{P}(X > 2)$ (M1) for valid approach

$= 1 - \mathrm{P}(X \leq 2)$ (A1) for correct value

$= 0.0525121075$

$= 0.0525$ A1 N4

[6]

2. (a) (i) Let X be the volume of a randomly selected milk soda.

The required probability

$= \mathrm{P}(X < 335)$ (M1) for valid approach

$= 0.0062096799$

$= 0.00621$ A1 N2

(ii) $\mathrm{P}(X > 330 \mid X < 335)$ (R1) for correct probability

$= \dfrac{\mathrm{P}(X > 330 \cap X < 335)}{\mathrm{P}(X < 335)}$ (A1) for correct formula

$= \dfrac{\mathrm{P}(330 < X < 335)}{\mathrm{P}(X < 335)}$

$= \dfrac{0.0057805633}{0.0062096799}$ (A1) for correct values

$= 0.9308955362$

$= 0.931$ A1 N4

[6]

(b) The required probability

$= 2 \times \mathrm{P}(X < 335) \times (1 - \mathrm{P}(X < 335))$ (M1) for valid approach

$= 2 \times 0.00620967 \times (1 - 0.00620967)$ (A1) for substitution

$= 0.01234222$

$= 0.0123$ A1 N3

[3]

(c) (i) The required expected number

$= 0.00620967 \times 60$ (A1) for correct formula

$= 0.3725802$

$= 0.373$ A1 N2

(ii) Let X : Number of required milk soda

$X \sim B(60, 0.00620967)$ (R1) for binomial distribution

The required probability

$= \mathrm{P}(X < 3)$ (M1) for valid approach

$= \mathrm{P}(X \leq 2)$ (A1) for correct value

$= 0.9937046328$

$= 0.994$ A1 N4

[6]

3. (a) (i) $\mathrm{P}(L < t) = 0.15$ (M1) for valid approach

$t = 66.89069986$

$t = 66.9$ A1 N2

(ii) $\mathrm{P}(L < 65 \mid L < t)$ (R1) for correct probability

$= \dfrac{\mathrm{P}(L < 65 \cap L < t)}{\mathrm{P}(L < t)}$ (A1) for correct formula

$= \dfrac{\mathrm{P}(L < 65)}{\mathrm{P}(L < t)}$

$= \dfrac{0.0477903304}{0.15}$ (A1) for correct values

$= 0.3186022024$

$= 0.319$ A1 N4

[6]

(b) The required probability

$= 2 \times \mathrm{P}(L < t) \times (1 - \mathrm{P}(L < t))$ (M1) for valid approach

$= 2 \times 0.15 \times (1 - 0.15)$ (A1) for substitution

$= 0.255$ A1 N3

[3]

(c) (i) The variance

$= 25 \times 0.15 \times (1 - 0.15)$ (A1) for correct formula

$= 3.1875$ A1 N2

(ii) $X \sim B(25, 0.15)$ (R1) for binomial distribution

The required probability

$= \mathrm{P}(X \geq 4)$ (M1) for valid approach

$= 1 - \mathrm{P}(X \leq 3)$ (A1) for correct value

$= 0.5288787147$

$= 0.529$ A1 N4

[6]

4. (a) (i) Let W be the weight of watermelons

$\mathrm{P}(W > t) = 0.1$ (M1) for valid approach

$t = 9.512620627$

$t = 9.51$ A1 N2

(ii) $\mathrm{P}(W < 9.8 \mid W > t)$ (R1) for correct probability

$= \dfrac{\mathrm{P}(W < 9.8 \cap W > t)}{\mathrm{P}(W > t)}$ (A1) for correct formula

$= \dfrac{\mathrm{P}(t < W < 9.8)}{\mathrm{P}(W > t)}$

$= \dfrac{0.0772500031}{0.1}$ (A1) for correct values

$= 0.772500031$

$= 0.773$ A1 N4

[6]

(b) The required probability

$= \mathrm{P}(W > t) \times \mathrm{P}(W > t) \times \mathrm{P}(W > t)$ (M1) for valid approach

$= 0.001$ A1 N2

[2]

(c) (i) The variance

$= 52 \times 0.1 \times (1 - 0.1)$ (A1) for correct formula

$= 4.68$ A1 N2

(ii) $X \sim B(52, 0.1)$ (R1) for binomial distribution

The required probability

$= \mathrm{P}(13 \le X \le 26)$ (M1) for valid approach

$= \mathrm{P}(X \le 26) - \mathrm{P}(X \le 12)$ (A1) for correct value

$= 0.0014868739$

$= 0.00149$ A1 N4

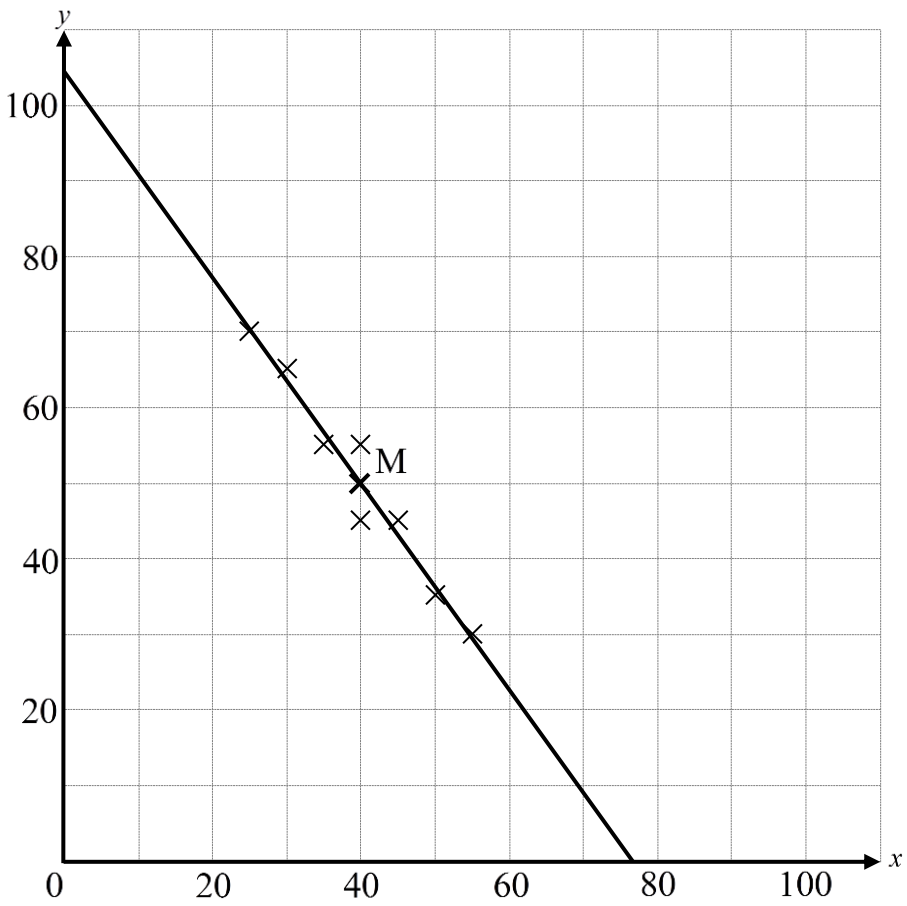
[6]

Chapter 22 Solution

Exercise 90

1. (a) For correct position A1
For labelling A1 N2

[2]



(b) For passing through M A1
For extending the line to the y-axis A1 N2

[2]

(c) Strong, negative A2 N2

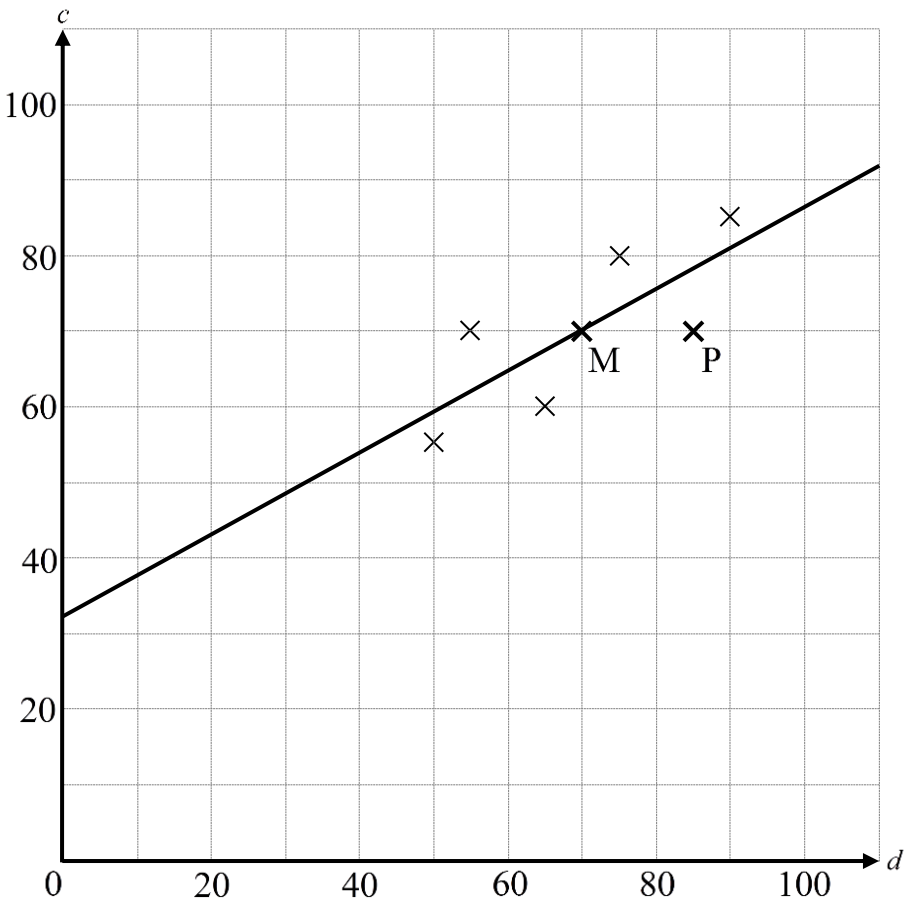
[2]

(d) 77 A2 N2

[2]

2. (a) For correct position A1
For labelling A1 N2

[2]



(b) (70, 70) A2 N2

[2]

(c) For correct position and labelling A1 N1

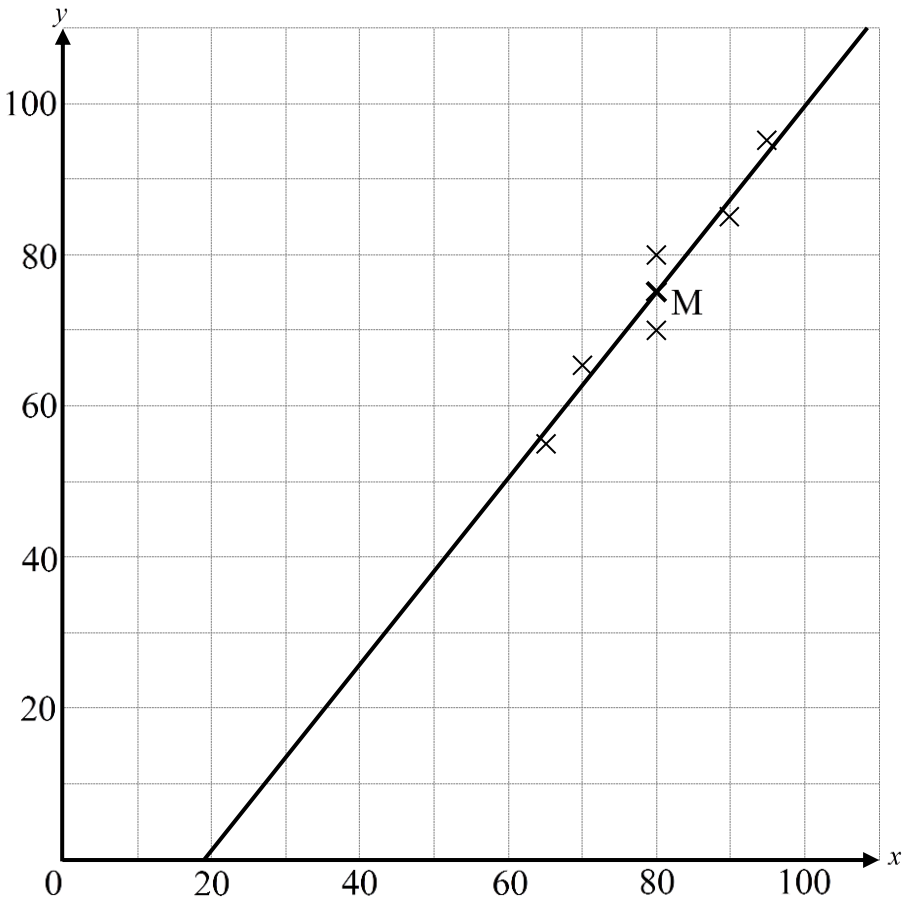
[1]

(d) For passing through M A1
For extending the line to the y-axis A1 N2

[2]

3. (a) For correct position A1
For labelling A1 N2

[2]



(b) For passing through M A1
For extending the line to the x-axis A1 N2

[2]

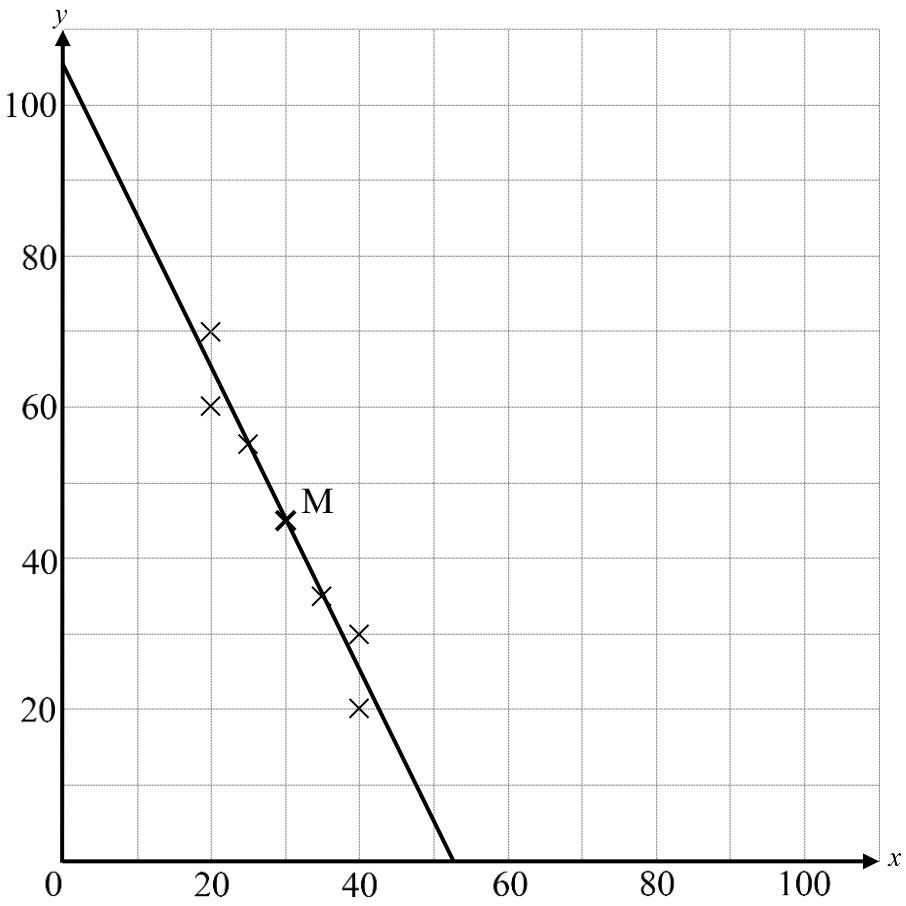
(c) $p = 65,\ q = 95$ A2 N2

[2]

4. (a) For passing through M A1

For extending the line to the y-axis A1 N2

[2]



(b) 15 A2 N2

[2]

(c) $p = 20$, $q = 40$ A2 N2

[2]

Exercise 91

1. (a) (i) $a = 0.2$ A1 N1

$b = 52.4$ A1 N1

(ii) The estimated final exam score

$= 0.2(85) + 52.4$ (A1) for substitution

$= 69.4$ A1 N2

[4]

(b) (i) $r = 0.1832541665$

$r = 0.183$ A1 N1

(ii) Weak, Positive A2 N2

[3]

2. (a) (i) $a = -2.085714286$

$a = -2.09$ A1 N1

$b = 96.0952381$

$b = 96.1$ A1 N1

(ii) The estimated temperature

$= -2.085714286(9) + 96.0952381$ (A1) for substitution

$= 77.32380953$

$= 77.3^{\circ}\text{C}$ A1 N2

[4]

(b) (i) $r = -0.6074200776$

$r = -0.607$ A1 N1

(ii) Moderate, Negative A2 N2

[3]

3. (a) (i) $a = 0.7121409922$

$a = 0.712$ A1 N1

$b = 7.222584856$

$b = 7.22$ A1 N1

(ii) The estimated Physics test score

$= 0.7121409922(25) + 7.222584856$ (A1) for substitution

$= 25.02610966$

$= 25.0$ A1 N2

[4]

(b) -0.989 A1 N1

[1]

(c) The estimated difference

$= 0.18(3)$ (M1) for valid approach

$= 0.54$ A1 N2

[2]

4. (a) (i) $r = -0.9565269783$

$r = -0.957$ A1 N1

(ii) $a = -0.7459677419$

$a = -0.746$ A1 N1

$b = 6.748252688$

$b = 6.75$ A1 N1

[3]

(b) 0.178 A1 N1

[1]

(c) The estimated average number of hours

$= 0.53(2.7)$ (A1) for substitution

$= 1.431$ hours A1 N2

[2]

Exercise 92

1. (a) (i) $a = 0.1566210046$

$a = 0.157$ A1 N1

$b = -5.752968037$

$b = -5.75$ A1 N1

(ii) a represents the average increase of university entrance mark when the public exam score is increased by 1. A1 N1

[3]

(b) The estimated university entrance mark

$= 0.1566210046(180) - 5.752968037$ (A1) for substitution

$= 22.43881279$

$= 22.4$ A1 N2

[2]

2. (a) (i) $a = 3.422857143$

$a = 3.42$ A1 N1

$b = 1.553333333$

$b = 1.55$ A1 N1

(ii) b represents the expected sales in 2011. A1 N1

[3]

(b) The estimated sales

$= 3.422857143(2.5) + 1.553333333$ (A1) for substitution

$= 10.11047619$

$= 10.1$ million dollars A1 N2

[2]

3. (a) (i) $a = 5.978021978$

$a = 5.98$ A1 N1

$b = 21.58241758$

$b = 21.6$ A1 N1

(ii) a represents the average increase of number of visitors when the maximum temperature is increased by 1 degree Celsius. A1 N1

b represents the expected number of visitors when the maximum temperature is zero degree Celsius. A1 N1

[4]

(b) The estimated number of visitors

$= 5.978021978(4) + 21.58241758$ (A1) for substitution

$= 45.49450549$

$= 45.5$ A1 N2

[2]

4. (a) (i) $a = 6.845588235$

$a = 6.85$ A1 N1

$b = 24.29338235$

$b = 24.3$ A1 N1

(ii) a represents the average increase of the hardness of a metal ingot when its breaking strength is increased by 1 tonne per cm. A1 N1

b represents the expected hardness of a metal ingot when its breaking strength is zero tonne per cm. A1 N1

[4]

(b) The estimated hardness

$= 6.845588235(6) + 24.29338235$ (A1) for substitution

$= 65.36691176$

$= 65.4$ A1 N2

[2]

Exercise 93

1. (a) (i) 5.5 A1 N1

(ii) 4.5 A1 N1

[2]

(b) $r_s = 0.0588235294$ (A1) for correct value

$r_s = 0.0588$ A1 N2

[2]

(c) There is a weak agreement between Ravi and Yannick. A1 N1

[1]

2. (a) The average equal ranks of the horse rider D and the horse rider E in Judge 1's ranking is 7.5. (A1) for correct value

The average equal ranks of the horse rider B and the horse rider C in Judge 2's ranking is 4.5. (A1) for correct value

$r_s = 0.8674698795$ (A1) for correct value

$r_s = 0.867$ A1 N4

[4]

(b) There is a strong agreement between the two judges. A1 N1

[1]

3. (a) $b = 2$, $d = 3$, $f = 5$ and $g = 4$

For any two correct answers A1

For all correct answers A1 N2

[2]

(b) $r_s = 0.9047619048$ (A1) for correct value

$r_s = 0.905$ A1 N2

[2]

(c) It is more likely for a team to be at a higher position in the league if it scores more goals. A1 N1

[1]

4. (a) $r_s = 0.0857142857$ (A1) for correct value

$r_s = 0.0857$ A1 N2

[2]

(b) (i) $a = 1,\ e = 5$ A2 N2

(ii) $R_s = 0.9$ A1 N1

(iii) The value of the Spearman's rank correlation coefficient increases. A1 N1

[4]

Exercise 94

1. (a) The values of the first variable are not quantifiable data. A1 N1

[1]

(b) $r_s = 0.8$ A2 N2

[2]

(c) It is more likely for a type of ice cream to have a higher ranking if its selling price is higher. A1 N1

[1]

2. (a) The values of the data are not linear. A1 N1

[1]

(b) $r_s = -0.8333333333$ (A1) for correct value

$r_s = -0.833$ A1 N2

[2]

(c) It is more likely to have a lower score in Geography test if the score in Chemistry test is higher. A1 N1

[1]

3. (a) The positions of teams in the league are not quantifiable data. A1 N1

[1]

(b) $p = 5.5,\ q = 7$ A2 N2

[2]

(c) $r_s = 0.9461247469$ (A1) for correct value

$r_s = 0.946$ A1 N2

[2]

(d) 1, 2, 3, 4 and 5 A1 N1

[1]

4. (a) $r = 0.8140243902$ (A1) for correct value

$r = 0.814$ A1 N2

[2]

(b) (i) $b = a - 2$ A1 N1

(ii) $g = a + 3$ A1 N1

[2]

(c) $r_s = 0.8928571429$ (A1) for correct value

$r_s = 0.893$ A1 N2

[2]

(d) The rank of the score of the diver B awarded by the trainee judge is unchanged. A1 N1

[1]

Exercise 95

1. (a) (i) $r = 0.8597409868$ (A1) for correct value

$r = 0.860$ A1 N2

(ii) $a = 0.0036032243$

$a = 0.00360$ A1 N1

$b = -0.6258602021$

$b = -0.626$ A1 N1

[4]

(b) The estimated monthly honey production

$= 0.0036032243(700) - 0.6258602021$ (A1) for substitution

$= 1.896396808$ (A1) for correct value

$= 1.9$ kg A1 N3

[3]

(c) The monthly honey production

$= 1.896396808 \times (1 + 2\%)^{12}$ (M1)(A1) for substitution

$= 1.896396808 \times 1.02^{12}$ (A1) for simplification

$= 2.405089691$

$= 2.41$ kg A1 N4

[4]

(d) $1.896396808 \times (1 + 2\%)^t = 3$ (M1) for setting equation

$1.896396808 \times 1.02^t - 3 = 0$ (A1) for simplification

By considering the graph of

$y = 1.896396808 \times 1.02^t - 3$, $t = 23.161402$. (A1) for correct value

Thus, the year is 2019. A1 N4

[4]

2. (a) (i) $r = 0.9822040739$ (A1) for correct value

$r = 0.982$ A1 N2

(ii) $a = 2.5625$ A1 N1

$b = 6.375$ A1 N1

[4]

(b) The estimated monthly honey production

$= 2.5625(24) + 6.375$ (A1) for substitution

$= 67.875$ (A1) for correct value

$= 68$ kg A1 N3

[3]

(c) The monthly consumption of chicken food

$= 67.875 \times (1 + 5\%)^6$ (M1)(A1) for substitution

$= 67.875 \times 1.05^6$ (A1) for simplification

$= 90.95899161$

$= 91.0$ kg A1 N4

[4]

(d) $67.875 \times (1 + 5\%)^t = 100$ (M1) for setting equation

$67.875 \times 1.05^t - 100 = 0$ (A1) for simplification

By considering the graph of

$y = 67.875 \times 1.05^t - 100$, $t = 7.9422239$. (A1) for correct value

Thus, the time is February 2019. A1 N4

[4]

3. (a) (i) $r = 0.9823629148$ (A1) for correct value

$r = 0.982$ A1 N2

(ii) $a = 14.06320542$

$a = 14.1$ A1 N1

$b = 188.3205418$

$b = 188$ A1 N1

[4]

(b) The estimated number of wolves

$= 14.06320542(11) + 188.3205418$ (A1) for substitution

$= 343.0158014$ (A1) for correct value

$= 343$ A1 N3

[3]

(c) $f(10) = 930$ (M1) for setting equation

$930 = 50(e^{0.01k(10)} + 2)$ (A1) for substitution

$930 - 50(e^{0.1k} + 2) = 0$

By considering the graph of $y = 930 - 50(e^{0.1k} + 2)$,

$k = 28.094027$.

$\therefore k = 28.1$ A1 N3

[3]

(d) $14.06320542t + 188.3205418$

$= 50(e^{0.01(28.094027)t} + 2)$ (M1) for setting equation

$14.06320542t + 188.3205418 = 50e^{0.28094027t} + 100$ (A1) for correct working

$50e^{0.28094027t} - 14.06320542t - 88.3205418 = 0$

By considering the graph of

$y = 50e^{0.28094027t} - 14.06320542t - 88.3205418$,

$t = 3.6593917$. (A1) for correct value

Thus, the year is 1984. A1 N4

[4]

4. (a) (i) $r = -0.925877311$ (A1) for correct value

$r = -0.926$ A1 N2

(ii) $a = -1.172413793$

$a = -1.17$ A1 N1

$b = 58.75862069$

$b = 58.8$ A1 N1

[4]

(b) The estimated number of breaths per minute

$= -1.172413793(12) + 58.75862069$ (A1) for substitution

$= 44.68965517$ (A1) for correct value

$= 45$ A1 N3

[3]

(c) $v(8) = 75$ (M1) for setting equation

$75 = \frac{10}{e^{8k}} + 70$ (A1) for substitution

$5 - \frac{10}{e^{8k}} = 0$

By considering the graph of $y = 5 - \frac{10}{e^{8k}}$,

$k = 0.0866434$.

$\therefore k = 0.0866$ A1 N3

[3]

(d) $\frac{10}{e^{0.0866434t}} + 70$ (M1) for setting equation

$= 1.5(-1.172413793t + 58.75862069)$

$\frac{10}{e^{0.0866434t}} + 70 = -1.75862069t + 88.13793104$ (A1) for correct working

$\frac{10}{e^{0.0866434t}} + 1.75862069t - 18.13793104 = 0$

By considering the graph of

$y = \frac{10}{e^{0.0866434t}} + 1.75862069t - 18.13793104$,

$t = 7.2902653$. (A1) for correct value

Thus, the time is after 7.29 minutes. A1 N4

[4]

Chapter 23 Solution

Exercise 96

1. (a) H_0: The number of heads follows the assigned distribution. A1 N1

[1]

(b) 10 A1 N1

[1]

(c) 3 A1 N1

[1]

(d) $\chi^2_{calc} = 6.933333333$ (A1) for correct value

$\chi^2_{calc} = 6.93$ A1 N2

[2]

(e) The null hypothesis is not rejected. A1

As $\chi^2_{calc} < 7.815$. A1 N2

[2]

2. (a) H_0: The last digits are evenly distributed. A1 N1

[1]

(b) 10 A1 N1

[1]

(c) 9 A1 N1

[1]

(d) 15.8 A2 N2

[2]

(e) The null hypothesis is not rejected. A1

As $\chi^2_{calc} < 16.919$. A1 N2

[2]

3. (a) H_0: The number of emails received on each day follows the assigned distribution. A1 N1

[1]

(b) $a = 4,\ b = 5$ A2 N2

[2]

(c) 5 A1 N1

[1]

(d) p-value $= 0.0907073916$ (A1) for correct value

p-value $= 0.0907$ A1 N2

[2]

(e) The null hypothesis is not rejected. A1

As p-value > 0.05. A1 N2

[2]

4. (a) H_0: The outcomes follows the assigned distribution. A1 N1

[1]

(b) 30 A1 N1

[1]

(c) 6 A1 N1

[1]

(d) p-value $= 0.00004963145972$ (A1) for correct value

p-value $= 0.0000496$ A1 N2

[2]

(e) The null hypothesis is rejected. A1

As p-value < 0.01. A1 N2

[2]

Exercise 97

1. (a) (i) H_0: The examination results and the corresponding sections are independent. A1 N1

(ii) H_1: The examination results and the corresponding sections are not independent. A1 N1

[2]

(b) 2 A1 N1

[1]

(c) $\chi^2_{calc} = 17.41714286$ (A1) for correct value

$\chi^2_{calc} = 17.4$ A1 N2

[2]

(d) The null hypothesis is rejected. A1

As $\chi^2_{calc} > 5.991$. A1 N2

[2]

2. (a) (i) H_0: The nationality of children and the choices of their most favourite fruits are independent. A1 N1

(ii) H_1: The nationality of children and the choices of their most favourite fruits are not independent. A1 N1

[2]

(b) 6 A1 N1

[1]

(c) $p\text{-value} = 4.56640566 \times 10^{-10}$ (A1) for correct value

$p\text{-value} = 4.57 \times 10^{-10}$ A1 N2

[2]

(d) The null hypothesis is rejected. A1

As $p\text{-value} < 0.01$. A1 N2

[2]

3. (a) H_0: The age of adults and their preferences are independent. A1 N1

[1]

(b) $x = 13,\ y = 39$ A2 N2

[2]

(c) 4 A1 N1

[1]

(d) $p\text{-value} = 4.702920665 \times 10^{-7}$ (A1) for correct value

$p\text{-value} = 4.70 \times 10^{-7}$ A1 N2

[2]

(e) The null hypothesis is rejected. A1

As $p\text{-value} < 0.05$. A1 N2

[2]

4. (a) H_0: The age of staffs and their number of investment bank accounts are independent. A1 N1

[1]

(b) $x = 5,\ y = 4$ A2 N2

[2]

(c) 8 A1 N1

[1]

(d) $p\text{-value} = 0.3261508802$ (A1) for correct value

$p\text{-value} = 0.326$ A1 N2

[2]

(e) The null hypothesis is not rejected. A1

As $p\text{-value} > 0.05$. A1 N2

[2]

Exercise 98

1. (a) The lengths of fishes are normally distributed. A1 N1

[1]

(b) (i) $H_0: \mu_1 = \mu_2$ A1 N1

(ii) $H_1: \mu_1 \neq \mu_2$ A1 N1

[2]

(c) p-value $= 0.0799994402$ (A1) for correct value

p-value $= 0.0800$ A1 N2

[2]

(d) The null hypothesis is not rejected. A1

As p-value > 0.05. A1 N2

[2]

2. (a) The volumes of bottles of milk are normally distributed. A1 N1

[1]

(b) (i) $H_0: \mu_D = \mu_L$ A1 N1

(ii) $H_1: \mu_D > \mu_L$ A1 N1

[2]

(c) p-value $= 0.0300468645$ (A1) for correct value

p-value $= 0.0300$ A1 N2

[2]

(d) The null hypothesis is rejected. A1

As p-value < 0.05. A1 N2

[2]

3. (a) $H_1: \mu_1 < \mu_2$ A1 N1

[1]

(b) p-value $= 0.0276305613$ (A1) for correct value

p-value $= 0.0276$ A1 N2

[2]

(c) -2.09 A1 N1

[1]

(d) The null hypothesis is rejected. A1

As p-value < 0.05. A1 N2

[2]

4. (a) $H_1: \mu_1 \neq \mu_2$ A1 N1

[1]

(b) p-value $= 0.9063680224$ (A1) for correct value

p-value $= 0.906$ A1 N2

[2]

(c) –0.120 A1 N1

[1]

(d) The null hypothesis is not rejected. A1

As p-value > 0.1. A1 N2

[2]

Exercise 99

1. (a) (i) H_0: The punctuality of buses and the locations of bus stops are independent. A1 N1

(ii) H_1: The punctuality of buses and the locations of bus stops are not independent. A1 N1

[2]

(b) 3 A1 N1

[1]

(c) $\chi^2_{calc} = 6.407563025$ (A1) for correct value

$\chi^2_{calc} = 6.41$ A1 N2

[2]

(d) The null hypothesis is rejected. A1

As $\chi^2_{calc} > 6.251$. A1 N2

[2]

(e) (i) The required probability

$= \frac{20}{100}$ (A1) for correct formula

$= \frac{1}{5}$ A1 N2

(ii) The required probability

$= \frac{1+3+8+3}{100}$ (A1) for correct formula

$= \frac{3}{20}$ A1 N2

(iii) The required probability

$= \frac{19}{19+17+20+29}$ (A1) for correct formula

$= \frac{19}{85}$ A1 N2

[6]

(f) The required probability

$= \left(\frac{85}{100}\right)\left(\frac{85-1}{100-1}\right)$ (A2) for correct formula

$= \frac{119}{165}$ A1 N3

[3]

2. (a) (i) H_0: The number of free lunches offered and the positions of staffs are independent. A1 N1

(ii) H_1: The number of free lunches offered and the positions of staffs are not independent. A1 N1

[2]

(b) 9 A1 N1

[1]

(c) $p\text{-value} = 5.946862571 \times 10^{-7}$ (A1) for correct value

$p\text{-value} = 5.95 \times 10^{-7}$ A1 N2

[2]

(d) 46.0 A1 N1

[1]

(e) The null hypothesis is rejected. A1

As $p\text{-value} < 0.05$. A1 N2

[2]

(f) (i) The required probability

$= \frac{4}{100}$ (A1) for correct formula

$= \frac{1}{25}$ A1 N2

(ii) The required probability

$= \frac{2+10+2+2+1+10+3+2}{100}$ (A1) for correct formula

$= \frac{8}{25}$ A1 N2

(iii) The required probability

$= \frac{2}{2+10+2+2}$ (A1) for correct formula

$= \frac{1}{8}$ A1 N2

[6]

(g) The required probability

$= \left(\frac{68}{100}\right)\left(\frac{68-1}{100-1}\right)$ (A2) for correct formula

$= \frac{1139}{2475}$ A1 N3

[3]

3. (a) (i) 8 A1 N1

(ii) 5.5 A1 N1

(iii) $1 \le X \le 5$ A1 N1

[3]

(b) (i) $a = 100$, $b = 60$ A2 N2

(ii) 6.5 A1 N1

(iii) 3.91 A1 N1

[4]

(c) The required probability

$= \frac{15+13}{200}$ (A1) for correct formula

$= \frac{7}{50}$ A1 N2

[2]

(d) (i) H_0: The number of dolls owned by a female student and her nationality are independent. A1 N1

(ii) H_1: The number of dolls owned by a female student and her nationality are not independent. A1 N1

[2]

(e) 4 A1 N1

[1]

(f) p-value $= 0.0623160411$ (A1) for correct value

p-value $= 0.0623$ A1 N2

[2]

(g) 8.95 A1 N1

[1]

(h) The null hypothesis is not rejected. A1

As p-value > 0.05. A1 N2

[2]

4. (a) (i) 25.5 A1 N1

(ii) 10.5 A1 N1

(iii) $1 \le X \le 10$ A1 N1

[3]

(b) (i) $a = 30,\ b = 146,\ n = 500$ A3 N3

(ii) 14.42 A1 N1

(iii) 8.25 A1 N1

[5]

(c) The required probability

$= \dfrac{75+125+65+15}{154+146}$ (A2) for correct formula

$= \dfrac{14}{15}$ A1 N3

[3]

(d) (i) H_0: The age of an interviewee and the number of yogurt parfait cups ate are independent. A1 N1

(ii) H_1: The age of an interviewee and the number of yogurt parfait cups ate are not independent. A1 N1

[2]

(e) 6 A1 N1

[1]

(f) $p\text{-value} = 1.443177288 \times 10^{-31}$ (A1) for correct value

$p\text{-value} = 1.44 \times 10^{-31}$ A1 N2

[2]

(g) 158 A1 N1

[1]

(h) The null hypothesis is rejected. A1

As $p\text{-value} < 0.01$. A1 N2

[2]

Exercise 100

1. (a) (i) $H_0: \mu_1 = \mu_2$ A1 N1

(ii) $H_1: \mu_1 < \mu_2$ A1 N1

[2]

(b) p-value $= 0.365277312$ (A1) for correct value

p-value $= 0.365$ A1 N2

[2]

(c) -0.350 A1 N1

[1]

(d) The null hypothesis is not rejected. A1

As p-value > 0.1. A1 N2

[2]

(e) The required probability

$= \frac{3}{3+5}$ (A1) for correct formula

$= \frac{3}{8}$ A1 N2

[2]

(f) (i) The required probability

$= \left(\frac{5}{9}\right)\left(\frac{6}{10}\right)$ (A1) for correct formula

$= \frac{1}{3}$ A1 N2

(ii) The required probability

$= 1 - \frac{1}{3}$ (A1) for correct formula

$= \frac{2}{3}$ A1 N2

[4]

2. (a) The number of times for a person to smoke in a day are normally distributed. A1 N1

[1]

(b) $H_1: \mu_1 > \mu_2$ A1 N1

[1]

(c) p-value $= 0.0055603914$ (A1) for correct value

p-value $= 0.00556$ A1 N2

[2]

(d) 3.11 A1 N1

[1]

(e) The null hypothesis is rejected. A1

As p-value < 0.01. A1 N2

[2]

(f) (i) The required probability

$= \frac{3}{12}$ (A1) for correct formula

$= \frac{1}{4}$ A1 N2

(ii) The required probability

$= \frac{3}{6+3}$ (A1) for correct formula

$= \frac{1}{3}$ A1 N2

[4]

(g) The required probability

$= \left(\frac{7}{12}\right)\left(\frac{7-1}{12-1}\right)\left(\frac{7-2}{12-2}\right)$ (A2) for correct formula

$= \frac{7}{44}$ A1 N3

[3]

3. (a) The scores in the assessments are normally distributed. A1 N1

[1]

(b) $H_1: \mu_{A1} \neq \mu_{B1}$ A1 N1

[1]

(c) $p\text{-value} = 0.0465127021$ (A1) for correct value

$p\text{-value} = 0.0465$ A1 N2

[2]

(d) 2.20 A1 N1

[1]

(e) The null hypothesis is rejected. A1

As $p\text{-value} < 0.05$. A1 N2

[2]

(f) (i) $\mu_{A1} < \mu_{A2}$ A1 N1

(ii) $p\text{-value} = 0.9003407897$ (A1) for correct value

As $p\text{-value} > 0.05$, the null hypothesis is not rejected. A2 N3

[4]

(g) The required probability

$= \left(\frac{5}{9}\right)\left(\frac{2}{7}\right) + \left(\frac{4}{9}\right)\left(\frac{5}{7}\right)$ (A2) for correct formula

$= \frac{10}{21}$ A1 N3

[3]

4. (a) The ball speeds of free kicks are normally distributed. A1 N1

[1]

(b) $H_1: \mu_{R1} < \mu_{R2}$ A1 N1

[1]

(c) $p\text{-value} = 0.0485601919$ (A1) for correct value

$p\text{-value} = 0.0486$ A1 N2

[2]

(d) -1.77 A1 N1

[1]

(e) The null hypothesis is rejected. A1

As $p\text{-value} < 0.1$. A1 N2

[2]

(f) (i) $\mu_{R2} \neq \mu_{M2}$ A1 N1

(ii) $p\text{-value} = 0.3962892074$ (A1) for correct value

As $p\text{-value} > 0.1$, the null hypothesis is not rejected. A2 N3

[4]

(g) The required probability

$= \left(\frac{2}{8}\right)\left(\frac{2}{9}\right) + \left(\frac{2}{8}\right)\left(\frac{4}{9}\right)$ (A2) for correct formula

$= \frac{1}{6}$ A1 N3

[3]

Alternative Solution for Question 96.3, 97.4, 99.1 and 99.2 With 5% Expected Values Lower Limit Considered

Exercise 96

3. (a) H_0: The number of emails received on each day follows the assigned distribution. A1 N1

[1]

(b) $a = 4,\ b = 5$ A2 N2

[2]

Group the first two columns together and the last two columns together:

Number of emails	0 - 1	2	3 - 5
Expected frequency	12	8	11
Observed frequency	8	6	17

(c) 2 A1 N1

[1]

(d) p-value $= 0.077845413$ (A1) for correct value

p-value $= 0.0778$ A1 N2

[2]

(e) The null hypothesis is not rejected. A1

As p-value > 0.05. R1 N2

[2]

Exercise 97

4. (a) H_0: The age of staffs and their number of investment bank accounts are independent. A1 N1

[1]

(b) $x = 5,\ y = 4$ A2 N2

[2]

Group the last four columns together:

		Number of investment bank accounts		
		Zero	One to Four	**Total**
Age	18 to 37	5	13	**18**
	38 to 57	4	12	**16**
	58 to 77	11	5	**16**
	Total	**20**	**30**	**50**

(c) 2 A1 N1

[1]

(d) p-value $= 0.0171562366$ (A1) for correct value

p-value $= 0.0172$ A1 N2

[2]

(e) The null hypothesis is rejected. A1

As p-value < 0.05. R1 N2

[2]

Exercise 99

1. (a) (i) H_0: The punctuality of buses and the locations of bus stops are independent. A1 N1

(ii) H_1: The punctuality of buses and the locations of bus stops are not independent. A1 N1

[2]

Group the first two columns together and the last two columns together:

		Bus stations	
		A and B	C and D
Arrivals	On time	36	49
	Late	4	11

(b) 1 A1 N1

[1]

(c) $\chi^2_{calc} = 1.307189542$ (A1) for correct value

$\chi^2_{calc} = 1.31$ A1 N2

[2]

(d) The null hypothesis is not rejected. A1

As $\chi^2_{calc} < 2.706$, the updated critical value . R1 N2

[2]

(e) (i) The required probability

$= \frac{20}{100}$ (A1) for correct formula

$= \frac{1}{5}$ A1 N2

(ii) The required probability

$= \frac{1+3+8+3}{100}$ (A1) for correct formula

$= \frac{3}{20}$ A1 N2

(iii) The required probability

$= \frac{19}{19+17+20+29}$ (A1) for correct formula

$= \frac{19}{85}$ A1 N2

[6]

(f) The required probability

$$= \left(\frac{85}{100}\right)\left(\frac{85-1}{100-1}\right)$$ (A2) for correct formula

$$= \frac{119}{165}$$ A1 N3

[3]

2. (a) (i) H_0: The number of free lunches offered and the positions of staffs are independent. A1 N1

(ii) H_1: The number of free lunches offered and the positions of staffs are not independent. A1 N1

[2]

Group the first two columns together and the last two columns together:

		Positions	
		Administration and Customer Service	Human Resources and Others
Number of free lunches	0	8	26
	1	9	25
	2	12	4
	3 or more	11	5

(b) 3 A1 N1

[1]

(c) p-value $= 0.0001608033962$ (A1) for correct value

p-value $= 0.000161$ A1 N2

[2]

(d) 20.1 A1 N1

[1]

(e) The null hypothesis is rejected. A1

As p-value < 0.05. R1 N2

[2]

(f) (i) The required probability

$= \frac{4}{100}$ (A1) for correct formula

$= \frac{1}{25}$ A1 N2

(ii) The required probability

$= \frac{2+10+2+2+1+10+3+2}{100}$ (A1) for correct formula

$= \frac{8}{25}$ A1 N2

(iii) The required probability

$= \frac{2}{2+10+2+2}$ (A1) for correct formula

$= \frac{1}{8}$ A1 N2

[6]

(g) The required probability

$= \left(\frac{68}{100}\right)\left(\frac{68-1}{100-1}\right)$ (A2) for correct formula

$= \frac{1139}{2475}$ A1 N3

[3]